I. Exponentialgleichungen

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1. Bestimme die Lösung in $G = \mathbb{R}$!

a) $e^x = 5$ b) $2 \cdot e^x = 5$ c) $e^{2x} = 5$ d) $e^{x-2} = 5$ e) $e^{2x-3} = 4$

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2. Bestimme die Lösung in $G = \mathbb{R}$!

a) $e^x = e^{x+1} - 1$ b) $e^x = e^{2x}$ c) $e^x = \frac{2}{e^x}$ d) $e^x = e^{\frac{1}{x}}$

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3. Bestimme die Lösung in $G = \mathbb{R}$!

a) $2 \cdot e^x + e^{x+2} = 10$ b) $e^{x-1} + 3 \cdot e^{x-2} = 10$ c) $2 \cdot e^x + 5 \cdot e^{x+1} = 6$

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4. Bestimme die Lösung in $G = \mathbb{R}$!

a) $\frac{4e^x}{e^x + 1} = 2$ b) $\sqrt{e^x - 1} = 2$ c) $(e^{\sqrt{x}} - 1)^2 = 1$

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5. Bestimme die Lösung in $G = \mathbb{R}$!

a) $e^{2x} - 10 \cdot e^x + 9 = 0$

b) $3 \cdot e^{2x} - 25 \cdot e^x + 8 = 0$

c) $\frac{4 \cdot e^x}{e^{2x} + 2} = 1$

d) $e^x + 1 = e^{-x}$

Lösungen

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1. a) $e^x = 5 \quad \Rightarrow \quad x = \ln 5$

b) $2 \cdot e^x = 5 \quad \Rightarrow \quad e^x = \frac{5}{2} \quad \Rightarrow \quad x = \ln\frac{5}{2}$

c) $e^{2x} = 5 \quad \Rightarrow \quad 2x = \ln 5 \quad \Rightarrow \quad x = \frac{1}{2} \cdot \ln 5$

d) $e^{x-2} = 5 \quad \Rightarrow \quad x - 2 = \ln 5 \quad \Rightarrow \quad x = 2 + \ln 5$

e) $e^{2x-3} = 4 \quad \Rightarrow \quad 2x - 3 = \ln 4 \quad \Rightarrow \quad 2x = 3 + \ln 4 \quad \Rightarrow$

$$x = \frac{3 + \ln 4}{2} = \frac{3}{2} + \frac{1}{2} \cdot n4 = \frac{3}{2} + \frac{1}{2} \cdot 2\ln 2 = \frac{3}{2} + \ln 2$$

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2. a) $e^x = e^{x+1} - 1 \quad \Rightarrow \quad e^x - e^{x+1} = -1 \quad \Rightarrow \quad e^x \cdot (1 - e) = -1 \quad \Rightarrow$

$$e^x = \frac{-1}{1-e} = \frac{1}{e-1} \quad \Rightarrow \quad x = \ln\frac{1}{e-1} = -\ln(e-1)$$

b) $e^x = e^{2x} \quad \Rightarrow \quad e^x - e^{2x} = 0 \quad \Rightarrow \quad e^x \cdot (1-e^x) = 0 \quad \Rightarrow \quad 1-e^x = 0 \quad \Rightarrow \quad x = 0$

c) $e^x = \frac{2}{e^x} \quad \Rightarrow \quad e^{2x} = 2 \quad \Rightarrow \quad 2x = \ln 2 \quad \Rightarrow \quad x = \frac{1}{2}\ln 2$

d) $e^x = e^{\frac{1}{x}} \quad \Rightarrow \quad x = \frac{1}{x} \quad \Rightarrow \quad x^2 = 1 \quad \Rightarrow \quad x = -1 \vee x = 1$

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3. a) $2 \cdot e^x + e^{x+2} = 10 \quad \Rightarrow \quad e^x \cdot (2+e^2) = 10 \quad \Rightarrow \quad e^x = \frac{10}{2+e^2} \quad \Rightarrow \quad x = \ln\frac{10}{2+e^2}$

b) $e^{x-1} + 3 \cdot e^{x-2} = 10 \quad \Rightarrow \quad e^{x-2} \cdot (e+3) = 10 \quad \Rightarrow \quad e^{x-2} = \frac{10}{e+3} \quad \Rightarrow$

$x - 2 = \ln\frac{10}{e+3} \quad \Rightarrow \quad x = 2 + \ln\frac{10}{e+3}$

c) $2 \cdot e^x + 5 \cdot e^{x+1} = 6 \quad \Rightarrow \quad e^x \cdot (1+5e) = 6 \quad \Rightarrow \quad e^x = \frac{6}{1+5e} \quad \Rightarrow \quad x = \ln\frac{6}{1+5e}$

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4. a) $\frac{4e^x}{e^x+1} = 2 \quad \Rightarrow \quad 4e^x = 2e^x + 2 \quad \Rightarrow \quad 2e^x = 2 \quad \Rightarrow \quad e^x = 1 \quad \Rightarrow \quad x = 0$

b) $\sqrt{e^x - 1} = 2 \quad \Rightarrow \quad e^x - 1 = 4 \quad \Rightarrow \quad e^x = 5 \quad \Rightarrow \quad x = \ln 5$

c) $(e^{\sqrt{x}} - 1)^2 = 1 \quad \Rightarrow \quad e^{\sqrt{x}} - 1 = -1 \vee e^{\sqrt{x}} - 1 = 1 \quad \Rightarrow \quad e^{\sqrt{x}} = 0 \vee e^{\sqrt{x}} = 2 \quad \Rightarrow$

$\sqrt{x} = 0 \vee \sqrt{x} = \ln 2 \quad \Rightarrow \quad x = 0 \vee x = (\ln 2)^2$

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5. a) $e^{2x} - 10 \cdot e^x + 9 = 0$

Substitution: $u := e^x$ und damit $u^2 - 10u + 9 = 0 \quad \Rightarrow \quad u = 1 \vee u = 9$

Resubstitution: $e^x = 1 \vee e^x = 9 \quad \Rightarrow \quad x = 0 \vee x = \ln 9 = 2 \cdot \ln 3$

b) $3 \cdot e^{2x} - 25 \cdot e^x + 8 = 0$

Substitution: $u := e^x$ und damit $3u^2 - 25u + 8 = 0 \quad \Rightarrow \quad u = 8 \vee u = \frac{1}{3}$

Resubstitution: $e^x = \frac{1}{3} \vee e^x = 8 \quad \Rightarrow \quad x = \ln\frac{1}{3} \vee x = \ln 8 \quad \Rightarrow$

$$x = -\ln 3 \ \vee \ u = -3\ln 2$$

c) $\frac{4 \cdot e^x}{e^{2x}+2} = 1 \quad \Rightarrow \quad 4 \cdot e^x = e^{2x}+2 \quad \Rightarrow \quad e^{2x} - 4e^x + 2 = 0$

Substitution: $u := e^x$ und damit $u^2 - 4u + 2 = 0 \quad \Rightarrow \quad u = 2-\sqrt{2} \ \vee \ u = 2+\sqrt{2}$

Resubstitution: $x = \ln(2-\sqrt{2}) \ \vee \ x = \ln(2+\sqrt{2})$

$$x = -\ln 3 \ \vee \ u = -3\ln 2$$

d) $e^x + 1 = e^{-x} \quad \Rightarrow \quad e^x + 1 = \frac{1}{e^x} \quad \Rightarrow \quad e^{2x} + e^x = 1 \quad \Rightarrow \quad e^{2x} + e^x - 1 = 0$

Substitution: $u := e^x$ und damit $u^2 + u - 2 = 0 \quad \Rightarrow \quad u = \frac{-1-\sqrt{5}}{2} \ \vee \ u = \frac{-1+\sqrt{5}}{2}$

Resubstitution: $x = \ln \frac{-1+\sqrt{5}}{2}$

II. Logarithmusgleichungen

1. Bestimme Definitions- und Lösungsmenge in $G = \mathbb{R}$!

a) $\ln(2x-1) = 3$ b) $2 \cdot \ln(x-1) = 3$ c) $\ln \frac{x}{x+1} = 1$

d) $\ln x + \ln(x+1) = 1$ e) $\ln(x+1) - \ln((1-x) = 2$ f) $\frac{2 \cdot \ln x}{1 - \ln x} = 1$

g) $(\ln x)^2 - 2 \cdot \ln x = 0$ h) $\sqrt{1 - \ln x} = 2$

Lösungen

1. a) Definitionsmenge: $2x - 1 > 0 \quad \Rightarrow \quad x > \frac{1}{2}$ und damit $D =]\frac{1}{2}; \infty[$

$\ln(2x-1) = 3 \quad \Rightarrow \quad 2x - 1 = e^3 \quad \Rightarrow \quad x = \frac{e^3+1}{2}$

b) Definitionsmenge: $x + 1 > 0 \quad \Rightarrow \quad x > -1$ und damit $D =]-1; \infty[$

$2 \cdot \ln(x+1) = 3 \quad \Rightarrow \quad \ln(x+1) = \frac{3}{2} \quad \Rightarrow \quad x + 1 = e^{1,5} \quad \Rightarrow \quad x = e^{1,5} - 1$

c) Definitionsmenge: $\frac{x}{x+1} > 0$

	$-\infty < x < -1$	$-1 < x < 0$	$x > 0$
x	$-$	$-$	$+$
$x+1$	$-$	$+$	$+$
$\frac{x}{x+1}$	$+$	$-$	$+$

und damit $D =]-\infty; 1[\cup]0; \infty[$

$$\ln\frac{x}{x+1} = 1 \quad \Rightarrow \quad \frac{x}{x+1} = e^1 = e \quad \Rightarrow \quad x = ex+1 \quad \Rightarrow \quad x - ex = 1 \quad \Rightarrow$$

$$x\cdot(1-e) = 1 \quad \Rightarrow \quad x = \frac{1}{1-e} \notin D$$

d) Definitionsmenge: $x > 0 \ \wedge \ x+1 > 0 \quad \Leftrightarrow \quad x > 0 \ \wedge \ x > -1$ und damit $D = \mathbb{R}^+$

$$\ln x + \ln(x+1) = 1 \quad \Rightarrow \quad \ln([x\cdot(x+1)] = 1 \quad \Rightarrow \quad x^2 + x = e \quad \Rightarrow \quad x^2 + x - e = 0$$

$$x = \frac{-1-\sqrt{1+4e}}{2} \ \vee \ x = \frac{-1+\sqrt{1+4e}}{2}$$

e) Definitionsmenge: $x+1 > 0 \ \wedge \ 1-x > 0 \quad \Leftrightarrow \quad x > -1 \ \wedge \ x < 1$ und damit $D =]-1; 1[$

$$\ln(x+1) - \ln((1-x) = 2 \quad \Rightarrow \quad \frac{x+1}{1-x} = e^2 \quad \Rightarrow \quad x+1 = e^2 - e^2x \quad \Rightarrow \quad x = \frac{e^2-1}{e^2+1}$$

f) $D = \mathbb{R}^+ \backslash \{e\}$

$$\frac{2\cdot\ln x}{1-\ln x} = 1 \quad \Rightarrow \quad 2\cdot\ln x = 1 - \ln x \quad \Rightarrow \quad 3\cdot\ln x = 1 \quad \Rightarrow \quad \ln = \frac{1}{3} \quad \Rightarrow \quad x = e^{\frac{1}{3}}$$

g) $D = \mathbb{R}^+$

$$(\ln x)^2 - 2\cdot\ln x = 0 \quad \Rightarrow \quad \ln x\cdot(\ln x - 2) = 0 \quad \Rightarrow \quad \ln x = 0 \ \vee \ \ln x - 2 = 0 \quad \Rightarrow$$

$$x = 1 \ \vee \ x = e^2$$

h) Definitionsmenge: $x > 0 \ \wedge 1 - x \leq e$ ergibt $D =]0; e]$

$$\sqrt{1-\ln x} = 2 \quad \Rightarrow \quad 1 - \ln x = 4 \quad \Rightarrow \quad \ln x = -3 \quad \Rightarrow \quad x = e^{-3}$$