Rechnen mit Wurzeln
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Alle Variablen stehen für postive Zahlen

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1. Radiziere teilweise

a) $2\sqrt{180} = 2\cdot\sqrt{36\cdot 5} = 2\cdot\sqrt{36}\cdot\sqrt{5} = 2\cdot 6\cdot\sqrt{5} = 12\sqrt{5}$

b) $\sqrt{176} = \sqrt{16\cdot 11} = 4\sqrt{11}$

c) $\sqrt{9000} = \sqrt{900\cdot 10} = 30\sqrt{10}$

d) $3\sqrt{507ab^2} = 3\sqrt{3\cdot 169ab^2} = 39b\sqrt{3a}$

e) $\sqrt{x^5} = \sqrt{x^4\cdot x} = x^2\sqrt{x}$

f) $\sqrt{5\cdot 10^5} = \sqrt{5\cdot 10\cdot 10^4} = 10^2\cdot\sqrt{50} = 5\cdot 10^2\cdot\sqrt{2} = 500\sqrt{2}$

g) $\sqrt{\frac{x^2+y^3}{8y^2}} = \sqrt{\frac{x^2+y^3}{2\cdot 4y^2}} = \frac{1}{2y}\cdot\sqrt{\frac{x^2+y^3}{2}}$

h) $\sqrt{18a^2+27b^2} = \sqrt{9\cdot(2a^2+3b^2)} = \sqrt{9}\cdot\sqrt{2a^2+3b^2} = 3\sqrt{2a^2+3b^2}$

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2. Ziehe unter die Wurzel

a) $7\sqrt{x} = \sqrt{49}\cdot\sqrt{x} = \sqrt{49x}$

b) $\frac{2}{3}\sqrt{a} = \sqrt{\frac{4}{9}a}$

c) $4a\sqrt{\frac{1}{2}b} = \sqrt{16a^2\cdot\frac{1}{2}b} = \sqrt{8a^2b}$

d) $-3x^2\sqrt{xy} = -\sqrt{9x^4\cdot xy} = -\sqrt{9x^5y}$

e) $3a\sqrt{\frac{1}{a}-3b} = \sqrt{9a^2}\cdot\sqrt{\frac{1}{a}-3b} = \sqrt{9a^2\cdot(\frac{1}{a}-3b)} = \sqrt{9a-27a^2b}$

f) $xy\sqrt{\frac{x}{y^3}} = \sqrt{x^2y^2\cdot\frac{x}{y^3}} = \sqrt{\frac{x^3}{y}}$

g) $\frac{2ab}{c}\cdot\sqrt{\frac{3c^3}{8a^2b}} = \sqrt{\frac{4a^2b^2}{c^2}\cdot\frac{3c^3}{8a^2b}} = \sqrt{1{,}5bc}$

h) $2x\sqrt{\frac{3x-1}{12x^3-4x^2}} = \sqrt{4x^2\cdot\frac{3x-1}{4x^2(3x-1)}} = 1$

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3. Mache den Nenner rational

a) $\frac{5\sqrt{10}}{\sqrt{8}} = \frac{5\sqrt{10}\cdot\sqrt{8}}{\sqrt{8}\cdot\sqrt{8}} = \frac{5\sqrt{80}}{8} = \frac{20\sqrt{5}}{8} = \frac{5}{2}\sqrt{5}$

b) $\frac{3\sqrt{7}}{4\sqrt{3}} = \frac{3\sqrt{7}\cdot\sqrt{3}}{4\sqrt{3}\cdot\sqrt{3}} = \frac{3\sqrt{21}}{4\cdot 3} = \frac{\sqrt{21}}{4}$

c) $\frac{81}{\sqrt{82}-1} = \frac{81\cdot(\sqrt{82}+1)}{(\sqrt{82}-1)\cdot(\sqrt{82}+1)} = \frac{81\cdot(\sqrt{82}+1)}{82-1} = \sqrt{82}+1$

d) $\frac{\sqrt{2}+\sqrt{3}}{3+\sqrt{6}} = \frac{(\sqrt{2}+\sqrt{3})\cdot(3-\sqrt{6})}{(3+\sqrt{6})\cdot(3-\sqrt{6})} = \frac{3\sqrt{2}-\sqrt{12}+3\sqrt{3}-\sqrt{18}}{9-6} =$

$= \frac{3\sqrt{2}-2\sqrt{3}+3\sqrt{3}-3\sqrt{2}}{9-6} = \frac{\sqrt{3}}{3}$

e) $\frac{4}{\sqrt{5}-\sqrt{3}} = \frac{4\cdot(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})\cdot(\sqrt{5}+\sqrt{3})} = \frac{4\cdot(\sqrt{5}+\sqrt{3})}{5-3} = 2\cdot(\sqrt{5}+\sqrt{3}) = 2\sqrt{5}+2\sqrt{3}$

f) $\frac{\sqrt{15}-\sqrt{13}}{\sqrt{15}+\sqrt{13}} = \frac{(\sqrt{15}-\sqrt{13})\cdot(\sqrt{15}-\sqrt{13})}{(\sqrt{15}+\sqrt{13})\cdot(\sqrt{15}-\sqrt{13})} = \frac{15-2\sqrt{195}+13}{15-13} = \frac{28-2\sqrt{195}}{2} = 14-\sqrt{195}$

g) $\frac{2\sqrt{6}}{2\sqrt{3}-\sqrt{2}} = \frac{2\sqrt{6}\cdot(2\sqrt{3}+\sqrt{2})}{(2\sqrt{3}-\sqrt{2})\cdot(2\sqrt{3}+\sqrt{2})} = \frac{4\sqrt{18}+2\sqrt{12}}{2^2\cdot 3-3} = \frac{12\sqrt{2}+4\sqrt{3}}{10} = 1{,}2\sqrt{2} + 0{,}4\sqrt{3}$

h) $\frac{a-b}{\sqrt{a}-\sqrt{b}} = \frac{(a-b)\cdot(\sqrt{a}+\sqrt{b})}{(\sqrt{a}-\sqrt{b})\cdot(\sqrt{a}+\sqrt{b})} = \frac{(a-b)\cdot(\sqrt{a}+\sqrt{b})}{a-b} = \sqrt{a}+\sqrt{b}$

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4. Multipliziere aus und vereinfache

a) $\left(\sqrt{8} - 3\sqrt{18}\right)^2 = \left(2\sqrt{2}-9\sqrt{2}\right)^2 = \left(-7\sqrt{2}\right)^2 = (-7)^2\cdot 2 = 98$

b) $\left(5\sqrt{2}+\sqrt{18}\right)^2 = \left(5\sqrt{2}+3\sqrt{2}\right)^2 = \left(8\sqrt{2}\right)^2 = 64\cdot 2 = 128$

c) $\left(2\sqrt{5}-\sqrt{18}\right)^2 = 2^2\cdot 5 - 2\cdot 2\sqrt{5}\cdot\sqrt{18} + 18 = 20 - 4\sqrt{90} + 18 = 38 - 12\sqrt{10}$

d) $\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{3}-\sqrt{2}\right) = 3 - 2 = 1$

e) $\left(\sqrt{20}-3\sqrt{2}\right)^2 - \left(3\sqrt{5}-\sqrt{8}\right)^2 = \left(20 - 2\cdot\sqrt{20}\cdot 3\sqrt{2} + 9\cdot 2\right) - \left(9\cdot 5 - 2\cdot 3\sqrt{20}\cdot\sqrt{8} + 8\right) =$

$= \left(38 - 6\sqrt{40}\right) - \left(53 - 6\sqrt{160} + 8\right) = 28 - 12\sqrt{10} - 63 + 12\sqrt{10} = -15$

f) $\left(2\sqrt{7}-3\sqrt{10}\right)\left(2\sqrt{7}-\sqrt{2}\right) = 4\cdot 7 - 2\sqrt{14} - 6\sqrt{70} - 3\sqrt{20} = 28 - 2\sqrt{14} - 6\sqrt{70} + 6\sqrt{5}$

g) $\left(2\sqrt{3} + 3\sqrt{6}\right)^2 = 4\cdot 3 + 12\sqrt{18} + 9\cdot 6 = 66 + 36\sqrt{2}$

h) $\sqrt{9+\sqrt{17}}\cdot\sqrt{9-\sqrt{17}} = \sqrt{(9+\sqrt{17})(9-\sqrt{17})} = \sqrt{81-17} = \sqrt{64} = 8$

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5. Vereinfache

a) $\frac{\sqrt{6}}{\sqrt{3}-1} - \frac{\sqrt{6}-\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{6}\cdot(\sqrt{3}+1)}{(\sqrt{3}-1)\cdot(\sqrt{3}+1)} - \frac{(\sqrt{6}-\sqrt{2})\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}} =$

$= \frac{\sqrt{18}+\sqrt{6}}{3-1} - \frac{\sqrt{18}-\sqrt{6}}{3} = \frac{3\sqrt{2}+\sqrt{6}}{2} - \frac{3\sqrt{2}-\sqrt{6}}{3} = \frac{3}{2}\sqrt{2} + \frac{1}{2}\sqrt{6} - \sqrt{2} + \frac{1}{3}\sqrt{6} =$

$= \frac{1}{2}\sqrt{2} - \frac{5}{6}\sqrt{6}$

b) $\frac{\sqrt{2}+1}{\sqrt{2}-\sqrt{3}} - \frac{\sqrt{3}-3}{\sqrt{6}} = \frac{(\sqrt{2}+1)\cdot(\sqrt{2}+\sqrt{3})}{(\sqrt{2}-\sqrt{3})\cdot(\sqrt{2}+\sqrt{3})} - \frac{(\sqrt{3}-3)\cdot\sqrt{6}}{\sqrt{6}\cdot\sqrt{6}} =$

$= \frac{2+\sqrt{6}+\sqrt{2}+\sqrt{3}}{2-3} - \frac{\sqrt{18}-3\sqrt{6}}{2} = \frac{2+\sqrt{6}+\sqrt{2}+\sqrt{3}}{-1} - \frac{3\sqrt{2}-3\sqrt{6}}{6} =$

$= -2 - \sqrt{6} - \sqrt{2} - \sqrt{3} - 0{,}5\sqrt{2} + 0{,}5\sqrt{6} = -2 - 1{,}5\sqrt{2} - \sqrt{3} - 0{,}5\sqrt{6}$

c) $= \left(2\sqrt{2}-\sqrt{3}\right)^2 - \frac{2\sqrt{2}-\sqrt{3}}{\sqrt{3}} = 4\cdot 2 - 2\cdot 2\sqrt{2}\cdot\sqrt{3} + 3 - \frac{(2\sqrt{2}-\sqrt{3})\cdot\sqrt{3}}{3} =$

$= 4 \cdot 2 - 2 \cdot 2\sqrt{2} \cdot \sqrt{3} + 3 - \dfrac{(2\sqrt{2} - \sqrt{3}) \cdot \sqrt{3}}{3} = 11 - 4\sqrt{6} - \dfrac{2\sqrt{6} - 3}{3} = 11 - 4\sqrt{6} - \dfrac{2}{3}\sqrt{6} + 1 =$

$= 12 - \dfrac{14}{3}\sqrt{6}$

6.

	Term	Definitionsmenge
a)	$T(x) = \sqrt{2x - 1}$	$D = [0{,}5; \infty[$
b)	$T(x) = 2\sqrt{1 - x}$	$D =]-\infty; 1]$
c)	$T(x) = \dfrac{1}{\sqrt{x + 1}}$	$D =]-1; \infty[$
d)	$T(x) = \sqrt{x^2 + 1}$	$D = \mathbb{R}$
e)	$T(x) = \sqrt{16 - x^2}$	$D =]-4; 4[$
f)	$T(x) = \sqrt{x} + \sqrt{-x}$	$D = \{0\}$
