Gleichungen

Für die Gleichung $x^n = a$ mit $a \in \mathbb{R}$ und $n \in \mathbb{N}$ mit $n \geq 2$ gilt

	n gerade	n ungerade
$a > 0$	$L = \{-\sqrt[n]{a}\,;\ \sqrt[n]{a}\}$	$L = \{\sqrt[n]{a}\}$
$a < 0$	$L = \{\ \}$	$L = \{-\sqrt[n]{\lvert a \rvert}\}$

Umgekehrt hat die Wurzelgleichung

$\sqrt[n]{x} = a$ mit $a \geq 0$ die Definitionsmenge $D = \mathbb{R}_0^+$ und die Lössungsmenge $L = \{a^n\}$

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1. Bestimme die Lösungen in $G = \mathbb{R}$.

a) $x^3 + 16 = 0$ b) $2x^5 - \frac{1}{16} = 0$ c) $0{,}5x^{-3} = 4$

d) $(1 - 3x)^4 = 25$ e) $x^{\frac{7}{2}} - 2 \cdot x^{\frac{3}{2}} = 0$ f) $\left(\frac{x}{x-1}\right)^3 = -27$

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2. Bestimme die Definitionsmenge D und die Lösungsmenge in $G = \mathbb{R}$.

a) $\sqrt[3]{3x - 1} = 2$ b) $\sqrt[4]{x^2 - 2} = 2$ c) $\sqrt[5]{2x^2 + 1} = 3$

d) $4 \cdot x^{\frac{2}{3}} + 1 = 2$ e) $x^{\frac{3}{4}} - 2 \cdot x^{\frac{3}{5}} = 0$ f) $(\frac{1}{2}x^{\frac{1}{2}} - 1)^{\frac{2}{3}} = 2$

Lösungen :

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2. a) $x^3 + 16 = 0 \quad \Rightarrow \quad x = -2\sqrt[3]{2}$

b) $2x^5 - \frac{1}{16} = 0 \quad \Rightarrow \quad x^5 = \frac{1}{32} \quad \Rightarrow \quad x = \frac{1}{2}$

c) $0{,}5x^{-3} = 4 \quad \Rightarrow \quad x^{-3} = 8 \quad \Rightarrow \quad \frac{1}{x^3} = 8 \quad \Rightarrow \quad x^3 = \frac{1}{8} \quad \Rightarrow \quad x = \frac{1}{2}$

d) $(1-3x)^4 = 25 \Rightarrow 1-3x = \sqrt[4]{25} = \sqrt{5} \Rightarrow -3x = \sqrt{5}-1 \Rightarrow x = \frac{1-\sqrt{5}}{3}$

e) $x^{\frac{7}{2}} - 2\cdot x^{\frac{3}{2}} = 0 \Rightarrow x^{\frac{3}{2}}\cdot(x^2-2) = 0 \Rightarrow x = 0 \vee x = \sqrt{2}$

f) $\left(\frac{x}{x-1}\right)^3 = -27 \Rightarrow \frac{x}{x-1} = -3 \Rightarrow x = -3x+3 \Rightarrow x = \frac{3}{4}$

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2. a) $3x-1 \geq 0 \Leftrightarrow x \geq \frac{1}{3} \Rightarrow D = [\frac{1}{3}; \infty[$

$\sqrt[3]{3x-1} = 2 \Rightarrow 3x-1 = 2^3 \Rightarrow 3x = 3 \Rightarrow x = 3$

b) $x^2-2 \geq 0 \Leftrightarrow x \leq -\sqrt{2} \vee x \geq \sqrt{2} \Rightarrow D =]-\infty; -\sqrt{2}] \cup [\sqrt{2}; \infty[$

$\sqrt[4]{x^2-2} = 2 \Rightarrow x^2-2 = 16 \Rightarrow x^2 = 18 \Rightarrow x = -3\sqrt{2} \vee x = 3\sqrt{2}$

c) $D = \mathbb{R}$

$\sqrt[5]{2x^2+1} = 3 \Leftrightarrow 2x^2+1 = 243 \Leftrightarrow x^2 = 121 \Leftrightarrow x = -11 \vee x = 11$

d) $D = \mathbb{R}_0^+$

$4\cdot x^{\frac{2}{3}}+1 = 2 \Leftrightarrow x^{\frac{2}{3}} = \frac{1}{4} \Rightarrow x^{\frac{1}{3}} = \sqrt{\frac{1}{4}} = \frac{1}{2} \Rightarrow x = \frac{1}{8}$

e) $D = \mathbb{R}_0^+$

$x^{\frac{3}{4}} - 2\cdot x^{\frac{3}{5}} = 0 \Leftrightarrow x^{\frac{3}{5}}\cdot(x^{\frac{3}{4}\cdot\frac{5}{3}}-2) = 0 \Leftrightarrow x = 0 \vee x^{\frac{5}{4}} = 2$

$\Leftrightarrow x = 0 \vee x = 2^{\frac{4}{5}} = \sqrt[5]{2^4} = \sqrt[5]{16}$

f) $\frac{1}{2}x^{\frac{1}{2}} - 1 \geq 0 \Leftrightarrow x^{\frac{1}{2}} \geq 2 \Rightarrow x \geq 4 \Rightarrow D = [4; \infty[$

$(\frac{1}{2}x^{\frac{1}{2}}-1)^{\frac{2}{3}} = 4 \Rightarrow \frac{1}{2}x^{\frac{1}{2}}-1 = 4^{\frac{3}{2}} = 8 \Rightarrow x^{\frac{1}{2}} = 18 \Rightarrow x = 324$
