6 Eigenschaften von Stammfunktionen und Integralen

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3 Integrale

a) $\int_{-2}^{1,2}(3x^2-x+2)dx = \left[x^3-\frac{1}{2}x^2+2x\right]_{-2}^{1,2} = \left(1,2^3-\frac{1}{2}\cdot 1,2^2+2,4\right) - \left(-8-2-4\right) = 17,408$

b) $\int_{1}^{5}3\cdot(\frac{5}{z}+2z)dz = \left[15\ln|z| \; + 3z^2\right]_{1}^{5} = \left(15\ln 5+75\right) - \left(0+3\right) = 15\ln 5+72$

c) $\int_{-4}^{4}\frac{1}{2}x\cdot(2+t)^2dx = \left[\frac{1}{4}x^2\cdot(2+t)^2\right]_{-4}^{4} = 0$

d) $\int_{2}^{4}\sqrt{2x+4}dx = \int_{2}^{4}(2x+4)^{\frac{1}{2}}dx = \left[\frac{1}{3}\cdot(2x+4)^{\frac{3}{2}}\right]_{2}^{4} = \frac{1}{3}\cdot\sqrt{12^3} - \frac{1}{3}\cdot\sqrt{8^3} = 8\sqrt{3} - \frac{16}{3}\sqrt{2}$

e) $\int_{4,5}^{3}(\frac{2}{t^2}+\frac{2}{t})dt = \left[-\frac{2}{t}+2\ln t\right]_{4,5}^{3} = \left(-\frac{2}{3}+2\ln 3\right) - \left(-\frac{4}{9}+2\ln 4,5\right) = -\frac{2}{9}+2\ln\frac{2}{3}$

f) $\int_{-3}^{-1}(-x^3+2e^x)dx = \left[-\frac{1}{4}x^4+2e^x\right]_{-3}^{-1} = \left(-\frac{1}{4}+2e^{-1}\right) - \left(-\frac{81}{4}+2e^{-3}\right) = 20+\frac{2}{e}-\frac{2}{e^3}$

g) $\int_{4}^{1}\frac{2+2z}{z^2+2z+1}dz = \left[\ln|z^2+2z+1|\right]_{4}^{1} = \ln 4-\ln 25 = \ln\frac{4}{25}$

h) $\int_{-1}^{0,5}6\cdot e^{4x-1}\cdot e^{2x}dx = \int_{-1}^{0,5}6\cdot e^{6x-1}dx = \left[e^{6x-1}\right]_{-1}^{0,5} = e^2-e^{-7}$

i) $\int_{1}^{3}(4u-2\ln u)du = \left[2u^2-2u\cdot\ln u+2u\right]_{1}^{3} = \left(18-6\ln 3+6\right) - \left(2-0+2\right) = 20-6\ln 3$

k) $\int_{1}^{\pi}(\frac{1}{x}+\sin x)dx = \left[\ln|x| \; - \cos x\right]_{1}^{\pi} = \left(\ln\pi-(-1)\right) - \left(\ln 1-\cos 1\right) = \ln\pi+\cos 1+1$

l) $\int_1^4 \frac{5t}{t^2+1}dt - \int_1^4 \frac{3t}{t^2+1}dt = \int_1^4 \frac{2t}{t^2+1}dt = \left[\ln(t^2+1)\right]_1^4 = \ln 17 - \ln 2 = \ln\frac{17}{2}$

m) $\int_0^{-2} \frac{2}{(1-x)^2}dx = \left[2\cdot(1-x)^{-1}\right]_0^{-2} = \frac{2}{3} - 2 = -\frac{4}{3}$

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4 Bestimmte Integrale

(1) $\int_1^2 (2x^3 - x^{-2})dx = \left[\frac{1}{2}x^4 + x^{-1}\right]_1^2 = \left(8 + \frac{1}{2}\right) - \left(\frac{1}{2} + 1\right) = 7$

(2) $\int_{-1}^{-2} \frac{2a}{4a^2+1}da = \left[\frac{1}{4}\ln(4a^2+1)\right]_{-1}^{-2} = \frac{1}{4}\ln 17 - \frac{1}{4}\ln 5 = \frac{1}{4}\ln\frac{17}{5}$

(3) $\int_0^2 x\cdot e^x dx \overset{\text{CAS}}{=} \left[(x-1)\cdot e^x\right]_0^2 = e^2 + 1$

(4) $\int_2^4 \frac{x+1}{x}dx = \int_2^4 (1 + \frac{1}{x})dx = \left[x + \ln|x|\right]_2^4 = \left(4 + \ln 4\right) - \left(2 + \ln 2\right) = 2 + \ln 2$

(5) $\int_2^4 \frac{\ln t}{t}dt = \left[\frac{1}{2}\cdot(\ln t)^2\right]_2^4 = \frac{1}{2}\cdot(\ln 4)^2 - \frac{1}{2}\cdot(\ln 2)^2 = \frac{1}{2}\cdot(\ln 4 - \ln 2)(\ln 4 + \ln 2) = \frac{1}{2}\cdot\ln 2\cdot\ln 8$

(6) $\int_{-2}^2 (u^2+1)(u^2-1)du = \int_{-2}^2 (u^4-1)du = \left[\frac{1}{5}u^5 - u\right]_{-2}^2 = 8{,}8$

(7) $\int_0^3 0{,}25r\cdot(r + e^2)dr = \left[\frac{1}{12}r^3 + e^2\cdot\frac{r^2}{8}\right]_0^3 = \frac{9}{4} + e^2\cdot\frac{9}{8} = \frac{9}{8}\cdot(2 + e^2)$

(8) $\int_0^2 2\sqrt{v}\cdot(1-v)dv = \int_0^2 (2v^{\frac{1}{2}} - 2v^{\frac{3}{2}})dv = \left[\frac{4}{3}v^{\frac{3}{2}} - \frac{4}{5}v^{\frac{5}{2}}\right]_0^2 = \frac{8}{3}\sqrt{2} - \frac{16}{5}\sqrt{2} = -\frac{8}{15}\sqrt{2}$

(9) $\int_{-1}^{-4} \frac{e^b}{2e^b+2}db = \left[\frac{1}{2}\ln(e^b+1)\right]_{-1}^{-4} = \frac{1}{2}\cdot\ln\left(\frac{e^{-4}+1}{e^{-1}+1}\right) = \frac{1}{2}\cdot\ln\left(\frac{e^4+1}{e^4+e^3}\right)$

(10) $\int_1^4 \sqrt{x^2-1}\,dx \overset{CAS}{=} \left[\frac{x\sqrt{x^2-1}}{2} - \frac{\ln(\sqrt{x^2-1}+x)}{2}\right]_1^4 = 2\sqrt{15} - \frac{\ln(\sqrt{15}+4)}{2}$

(11) $\int_2^e 20\cdot(x-1)\cdot\ln(x-1)\,dx = 20\cdot\left[(\frac{x^2}{2} - x + \frac{1}{2})\cdot\ln(x-1) - \frac{x\cdot(x-2)}{4}\right]$

$= (10e^2 - 20e + 10)\cdot\ln(e-1) - 5e\cdot(e-2)$

(12) $\int_{-2}^2 (2x^3 + x)\,dx = 0$

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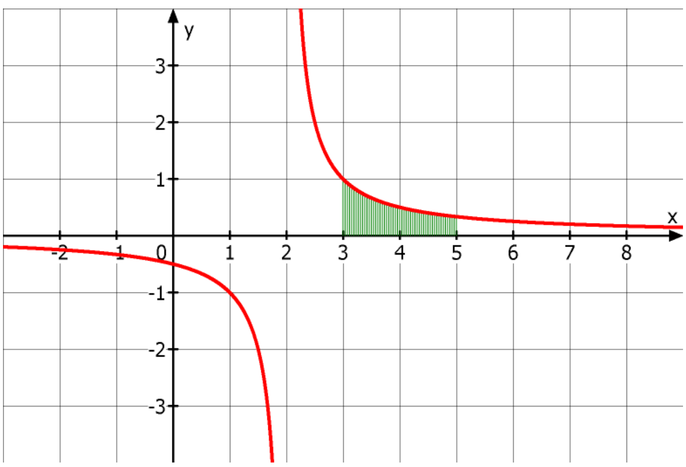
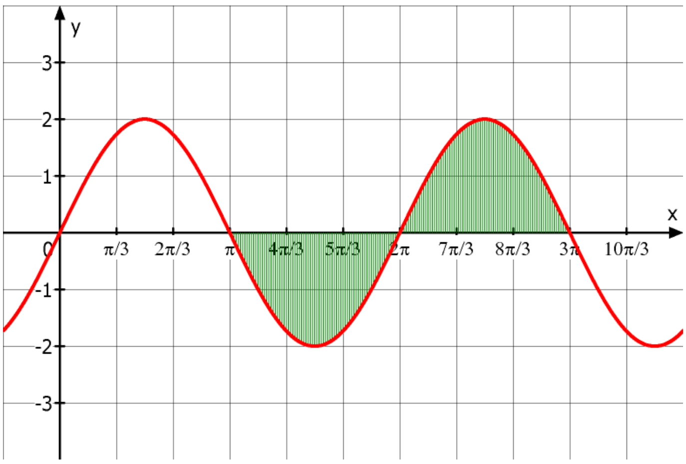
5 CAS

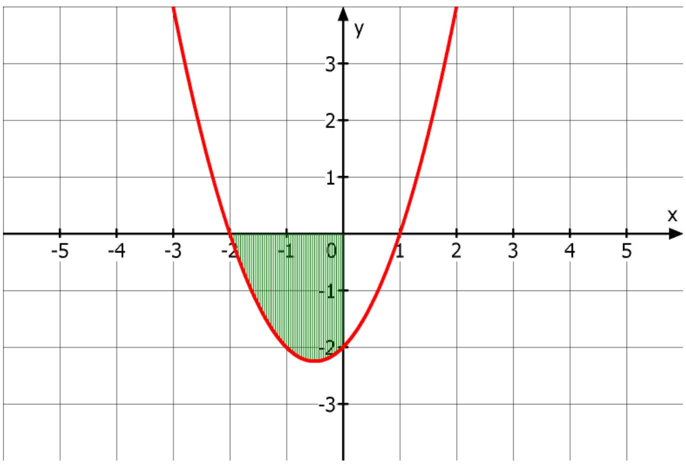
a) $\int_3^5 \frac{1}{x-2}\,dx = \ln 3$



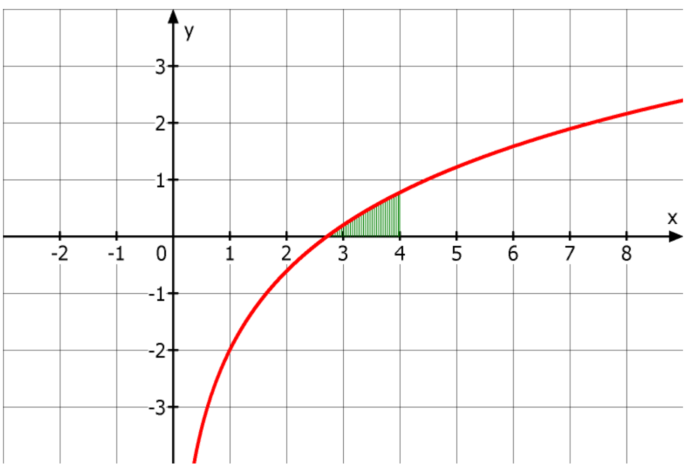
b) $\int_\pi^{3\pi} 2\cdot\cos(t - \frac{\pi}{2})\,dt = \left[2\cdot\sin(t - \frac{\pi}{2})\right]_\pi^{3\pi} = 2 - 2 = 0$



c) $\int_{-2}^{0}(x-1)(2+x)dx = \int_{-2}^{0}(x^2+x-2)dx = \left[\frac{1}{3}x^3+\frac{1}{2}x^2-2x\right]_{-2}^{0} = -(-\frac{8}{3}+2+4) = -\frac{10}{3}$



d) $\int_{e}^{4}(2\ln u-2)du = \left[2u\ln u-4u\right]_{e}^{4} = (8\cdot\ln 4-16) - (2e-4e) = 16\cdot\ln 2-16+2e$



6 Unbestimmte Integrale und Integrationsgrenzen

a) $\int 2^{2-2t}dt = -\frac{1}{2}e^{2-2t}+C$

b) $\int 4\cdot\cos(4x)dx = \sin(4x)+C$

c) $\int_{-a}^{a} x^2dx = 10 \quad \Rightarrow \quad \frac{2}{3}a^3 = 10 \quad \Rightarrow \quad a = \sqrt[3]{15}$

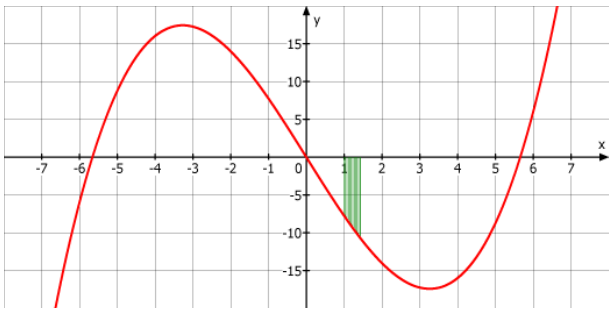
d) $\int \frac{(t-1)(t+1)}{2t^3-2t}dt = \frac{1}{2}\ln|2t-2| + C$

e) $\int 6\cdot\sin(3t+3)st = -2\cdot\cos(t+3) + C$

f) $\int_{1}^{a} (\frac{1}{4}x^3-8x)dx = -4 \quad \Leftrightarrow \quad \frac{1}{16}a^4-4a^2-\frac{1}{16}+4 = -4 \quad \Leftrightarrow \quad a^4-64a^2+127 = 0$

$a^2 = 32 \pm \sqrt{897}$

$\Rightarrow \quad a = -\sqrt{32-\sqrt{897}} \lor a = \sqrt{32-\sqrt{897}} \lor a = -\sqrt{32+\sqrt{897}} \lor a = \sqrt{32+\sqrt{897}}$



g) $\int_{-2}^{a} (-\frac{2}{u^2}+2u)du = -3 \quad \Rightarrow \quad \frac{2}{a}+a^2+1-4 = -3 \quad \Rightarrow \quad a = -\sqrt{2}$

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7 Achsensymmetrie

a) $\int_{-3}^{3} x^2dx = \left[\frac{1}{3}x^3\right]_{-3}^{3} = 9-(-9) = 18 = 2\cdot\int_{0}^{3} x^2dx$

Zur Begründung vgl. Überschrift.

b) $f(x) = x^4$, $g(x) = -x^2+4$ und $h(x) = x^{2012}$

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8 Graph

a) Ansatz: $f(x) = ax\cdot(x^2-9)$

$f(1) = -1 \quad \Rightarrow \quad 1 = a\cdot(-8) \quad \Rightarrow \quad a = \frac{1}{8}$

b) $f(-x) = -f(x)$

c) Ergibt sich aus der Punktsymmetrie.

d) $\int_{-1}^{2} f(x)dx = \int_{-1}^{1} f(x)dx + \int_{1}^{2} f(x)dx = \int_{1}^{2} f(x)dx$

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9 Integralfunktionen und Stammfunktionen

$f(x) = -\frac{6}{x^2} + 3\cdot(x^2+1) \quad \Rightarrow \quad F(x) = \frac{6}{x} + x^3 + 3x + C$

a) $\int_{2}^{x} f(t)dt = \left[\frac{6}{t} + t^3 + 3t\right]_2^x = \frac{6}{x} + x^3 + 3x - 17$

b) $F(x) = \frac{6}{x} + x^3 + 3x$

c) $f(x) = 0 \quad \Rightarrow \quad x = 1$

10 ist Minimum von F. F hat damit keine Nullstelle.

d) $C \leq -10$

G 12

	L	$\overline{L}$	
B	0,05		p
$\overline{B}$	0,20		$1-p$
	0,25	0,75	

$0{,}25\cdot p = 0{,}05 \quad \Rightarrow \quad p = 0{,}2$

Es müssen 40 Fahrräder Mängel an den Bremsen haben.