4 Der Hauptsatz der Differential- und Integralrechnung

2 Stammfunktion und Integral

a) $\int_0^4 2x\,dx = \left[x^2\right]_0^4 = 16$

b) $\int_0^4 -u\,du = \left[-\frac{1}{2}u^2\right]_0^4 = -8$

c) $\int_1^{-4} kx\,dx = \left[\frac{1}{2}k\cdot x^2\right]_1^{-4} = \frac{1}{2}k\cdot 16 - \frac{1}{2}k\cdot 1 = 7{,}5k$

d) $\int_0^2 3x^2dx = \left[x^3\right]_0^2 = 8$

e) $\int_{-2}^4 kt^2dt = \left[k\cdot\frac{t^3}{3}\right]_{-2}^4 = k\cdot\frac{64}{3} - k\cdot(-\frac{8}{3}) = 24k$

f) $\int_1^5 k\,dx = \left[kx\right]_1^5 = 5k - k = 4k$

g) $\int_{-2}^2 ax^3dx = \left[a\cdot\frac{x^4}{4}\right]_{-2}^2 = 0$

h) $\int_{-1}^1 (2x-1)dx = \left[x^2 - x\right]_{-1}^1 = \left(1-1\right) - \left(1+1\right) = -2$

i) $\int_{-2}^{-1} \frac{1}{3}v^2dv = \left[\frac{1}{9}v^3\right]_{-2}^{-1} = -\frac{1}{9} - \left(-\frac{8}{9}\right) = \frac{7}{9}$

k) $\int_3^{-3} 3x^5dx = \left[\frac{1}{2}x^6\right]_3^{-3} = 0$

$$\text{l)}\ \int_{10}^{20} dx = \left[x \right]_{10}^{20} = 20 - 10 = 0$$

$$\text{m)}\ \int_{-1}^{1} k^4 t^2 dt = \left[k^4 \cdot \frac{t^3}{3} \right]_{-1}^{1} = \frac{1}{3}k^4 - \left(-\frac{1}{3}k^4 \right) = \frac{2}{3}k^4$$

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3 Korrektur

$$\int_{-2}^{-1} -2x dx = \left[-x^2 \right]_{-2}^{-1} = -(-1)^2 - [-(-2)^2] = 3$$

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4 Prozentualer Flächenvergleich

$$A_1 = \int_{0,5}^{2} (2 - \frac{1}{2}x^2) dx = \left[2x - \frac{1}{6}x^3 \right]_{0,5}^{2} = \left(4 - \frac{8}{6} \right) - \left(1 - \frac{1}{6} \right) = \frac{11}{6}$$

$$\int_{2}^{3} (2 - \frac{1}{2}x^2) dx = \left[2x - \frac{1}{6}x^3 \right]_{2}^{3} = \left(6 - \frac{27}{6} \right) - \left(4 - \frac{8}{6} \right) = -\frac{7}{6} \quad \Rightarrow \quad A_2 = \frac{7}{6}$$

$$\frac{A_1 - A_2}{A_1} = \frac{\frac{11}{6} - \frac{7}{6}}{\frac{7}{6}} = \frac{4}{7} \approx$$

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5 Fläche

$$\text{a)}\ \frac{6}{25}x^2 = 4 - x \quad \Rightarrow \quad x = \frac{5}{2} \ \vee \ x = -\frac{20}{3}$$

$$\int_{0}^{2,5} \frac{6}{25}x^2 dx = \left[\frac{2}{25}x^3 \right]_{0}^{2,5} = \frac{5}{4} \text{ und damit A } = \frac{5}{4} + \frac{1}{2} \cdot 1,5^2 = \frac{19}{8}$$

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6 Stammfunktionen und Integral

$$\text{a)}\ F(x)\ ) = 2x^3 + 4x + 1 \quad \Rightarrow \quad F'(x) = 6x^2 + 4$$

$$\int_{1}^{3} (6x^2 + 4) = \left[2x^3 + 4x \right]_{1}^{3} = \left(54 + 12 \right) - \left(2 + 4 \right) = 60$$

$$\text{b)}\ F(x) = -\frac{1}{4}x^4 + 1,5x^2 \quad \Rightarrow \quad F'(x) = -x^3 + 3x$$

$$\int_{-4}^{4}(-x^3+3x)dx = \left[-\frac{1}{4}x^4+1{,}5x^2\right]_{-4}^{4} = 0$$

c) $F(x) = -x+\frac{2}{x} = -x+2x^{-1} \Rightarrow F'(x) = -1-2x^{-2} = -1-\frac{2}{x^2}$

$$\int_{-2}^{-1}(-1-\frac{2}{x^2})dx = \left[-x+\frac{2}{x}\right]_{-2}^{-1} = \left(1-2\right) - \left(2-1\right) = -2$$

d) $F(t) = \frac{2t}{2t+1} \Rightarrow F'(t) = \frac{2\cdot(2t+1) - 2t\cdot 2}{(2t+1)^2} = \frac{2}{(2t+1)^2}$

$$\int_{0}^{4}\frac{2}{(2t+1)^2}dt = \left[\frac{2t}{2t+1}\right]_{0}^{4} = \frac{8}{9}-2 = -\frac{10}{9}$$

e) $F(x) = x^2\cdot\sin(-x) \Rightarrow F'(x) = 2x\cdot\sin(-x) + x^2\cdot\cos(-x)\cdot(-1) =$

$= x\cdot[2\cdot\sin(-x) - x\cdot\cos(-x)]$

$$\int_{-2}^{2}x\cdot[\sin(-x)-x\cdot\cos(-x)]dx = \left[x^2\cdot\sin(-x)\right]_{-2}^{2} = 0$$

f) $F(u) = \sqrt{0{,}5u^2+u} \Rightarrow F'(u) = \frac{1}{2\cdot\sqrt{0{,}5u^2+u}}\cdot(u+1) = \frac{u+1}{2\cdot\sqrt{0{,}5u^2+u}}$

$$\int_{3}^{5}\frac{u+1}{2\cdot\sqrt{0{,}5u^2+u}}du = \left[\sqrt{0{,}5u^2+u}\right]_{3}^{5} = \frac{1}{2}\sqrt{70}-\frac{1}{2}\sqrt{30}$$

g) $F(x) = (x-2x^2)^2 \Rightarrow F'(x) = 2\cdot(x-2x^2)\cdot(1-4x) = 16x^3-12x^2+2x$

$$\int_{2}^{-1}(16x^3-12x^2+2x) = \left[(x-2x^2)^2\right]_{2}^{-1} = 9-36 = -27$$

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7 Integrale

a) $\int_{-4}^{2}\frac{1}{4}xdx = \left[\frac{1}{8}x^2\right]_{-4}^{2} = \frac{1}{2} - 2 = -1{,}5$

b) $\int_{-1}^{2}(-2u+2)du = \left[-u^2+2u\right]_{-1}^{2} = \left(-4+4\right) - \left(-1-2\right) = 3$

c) $\int_{1}^{-2}(1{,}5t+0{,}5)dt = \left[\frac{3}{4}t^2+0{,}5t\right]_{1}^{-2} = \left(3-1\right) - \left(\frac{3}{4}+0{,}5\right) = 0{,}75$

d) $\int_{-2}^{2{,}5}(-x^2+2)dx = \left[-\frac{1}{3}x^3+2x\right]_{-2}^{2{,}5} = \left(-\frac{125}{24}+10\right) - \left(\frac{8}{3}-4\right) = 6\frac{1}{8}$

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8 Integralfunktionen

a) $f: x \to 3x^2 \; ; \; a = -2 \quad \Rightarrow \quad I_a(x) = \int_{a}^{x} f(t)dt = \int_{-2}^{x} 3t^2 dt = \left[t^3\right]_{-2}^{x} = x^3+8$

Nullstellen von I_a: $x = -2$

Extremstellen von I_a: keine

b) $f: x \to x^3 \; ; \; a = 2 \quad \Rightarrow \quad I_a(x) = \int_{a}^{x} f(t)dt = \int_{2}^{x} t^3 dt = \left[\frac{1}{4}t^4\right]_{2}^{x} = \frac{1}{4}x^4-4$

Nullstellen von I_a: $x = -2 \vee x = 2$

Extremstellen von I_a: $x = 0$

c) $f: x \to -8x+1 \; ; \; a = -0{,}5 \quad \Rightarrow \quad I_a(x) = \int_{a}^{x} f(t)dt = \int_{-0{,}5}^{x} (-8t+1)dt =$

$= \left[-4t^2+t\right]_{-0{,}5}^{x} = \left(-4x^2+x\right) - \left(-1-0{,}5\right) = -4x^2+x+1{,}5$

Nullstellen von I_a: $x = -0{,}5 \vee x = \frac{3}{4}$

Extremstellen von I_a: $x = \frac{1}{8}$

d) $f: x \to 3x^2-2x \; ; \; a = 2 \quad \Rightarrow \quad I_a(x) = \int_{a}^{x} f(t)dt = \int_{2}^{x} (3t^2-2t)dt = \left[t^3-t^2\right]_{2}^{x} =$

$= \left(x^3 - x^2\right) - \left(8 - 4\right) = x^3 - x^2 - 4$

Nullstellen von I_a: $x = 2$

$(x^3 - x^2 - 4):(x - 2) = x^2 - x + 2$

Extremstellen von I_a: $x = 0 \vee x = \frac{2}{3}$keine

e) $f : x \rightarrow 3\cdot(x+1)(x-1) = 3x^2 - 3$; $a = -1 \Rightarrow I_a(x) = \int_a^x f(t)dt = \int_{-1}^x (3t^2 - 3)dt =$

$= \left[t^3 - 3t\right]_{-1}^x = \left(x^3 - 3x\right) - \left(-1 + 3\right) = x^3 - 3x - 2$

Nullstellen von I_a: $x = 2$

$(x^3 - 3x - 2):(x + 1) = x^2 - x - 2$

$x^2 - x - 2 = 0 \Leftrightarrow x = -1 \vee x = 2$

Extremstellen von I_a: $x = -1 \vee x = 1$

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11 Freier Fall

$$\int_0^3 9{,}81t\,dt = \left[9{,}81\cdot\frac{1}{2}t^2\right]_0^3 = 44{,}125 \ \left(m\right)$$

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9 Bestimmung bestimmter Integrale

	a)	b)	c)	d)
Integral	$2 - 1 = 1$	$-0{,}2 - (-1{,}6) = 1{,}4$	$1{,}7 - 0{,}7 = 1$	$0{,}3 - 1 = -0{,}7$
Fläche	1	1,4	1	1,1

10

a) $f'(x) = \frac{8}{x^3}$

$$f(8) - f(3) = \int_3^8 \frac{8}{x^3}\,dx = \left[-\frac{4}{x^2}\right]_3^8 = \frac{7}{36}$$

b) $f(x) = -\frac{4}{x^2} + C \quad \Rightarrow \quad -20 = -\frac{4}{(-0{,}5)^2} + C \quad \Rightarrow \quad C = 36 \quad \Rightarrow \quad f(10) = 35{,}96$

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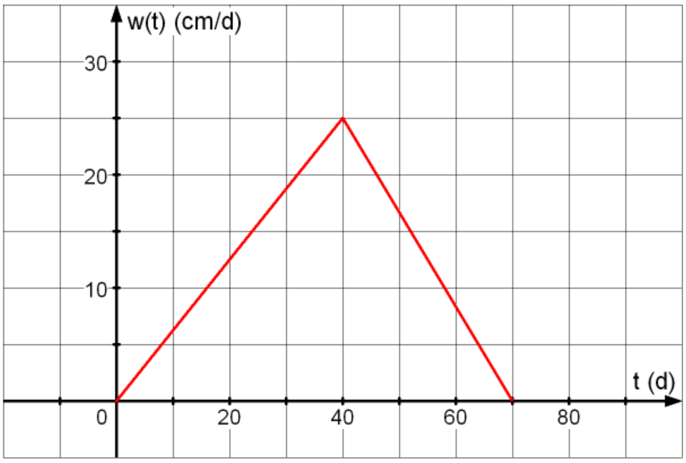
12 Hopfenwachstum



$$\Delta L = \frac{1}{2} \cdot 40\text{ d} \cdot 25\frac{\text{cm}}{\text{d}} + \frac{1}{2} \cdot 30\text{ d} \cdot 25\frac{\text{cm}}{\text{d}} = 875\text{ cm} = 8{,}75\text{ m}$$

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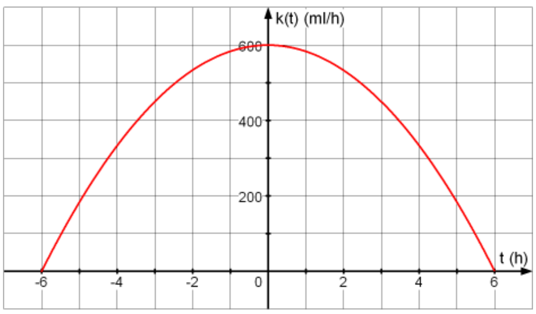
13 Photosynthese

a)



b) $\int_0^6 (600 - \frac{50}{3}t^2)\,dt = \left[600t - \frac{50}{9}t^3\right]_0^6 = 3600 - 1200 = 2400$

Verbrauch pro 1 m^2: 4800 ml

Gesamtfläche: $200000 \cdot 25\text{cm}^2 = 500\text{ m}^2$

Verbrauch eines Baumes: 2400 Liter

G 14

a) $f(x) = (3+x^4)^2 \quad \Rightarrow \quad f'(x) = 2\cdot(3+x^4)\cdot 4x^3 = 8x^3\cdot(3+x^4)$

b) $f(x) = x^{0,3} \quad \Rightarrow \quad f'(x) = 0,3\cdot x^{-0,7} = \frac{0,3}{x^{0,7}} = \frac{3}{10\cdot \sqrt[10]{x7}}$

c) $f(x) = \sqrt[4]{x^7} = x^{\frac{7}{4}} \quad \Rightarrow \quad f'(x) = \frac{7}{4}\cdot x^{\frac{3}{4}} = \frac{7}{4}\cdot \sqrt[4]{x^3}$

d) $g(x) = x^{-3} + \cos\frac{1}{x} \quad \Rightarrow \quad g'(x) = -3\cdot x^{-4} - \sin\frac{1}{x}\cdot(-\frac{1}{x^2}) = \frac{x^2\cdot \sin\frac{1}{x} - 3}{x^4}$

G 15

Wenn $\left(\vec{a} \times \vec{b}\right) \times \vec{c} = 0$ ist, dann ist einer der drei Vektoren der Nullvektor

oder

die drei Vektoren sind parallel zu einer Ebene.

G 16

a) $\cos\alpha = \cos 540° = -1$ mit $0° < \alpha < 360°$ bedeutet $\alpha = 180°$.

b) $\sin\alpha = \sin 720° = 0$ mit $0° < \alpha < 360°$ bedeutet $\alpha = 180°$.
