Das Integral als Flächenbilanz; die Integralfunktion

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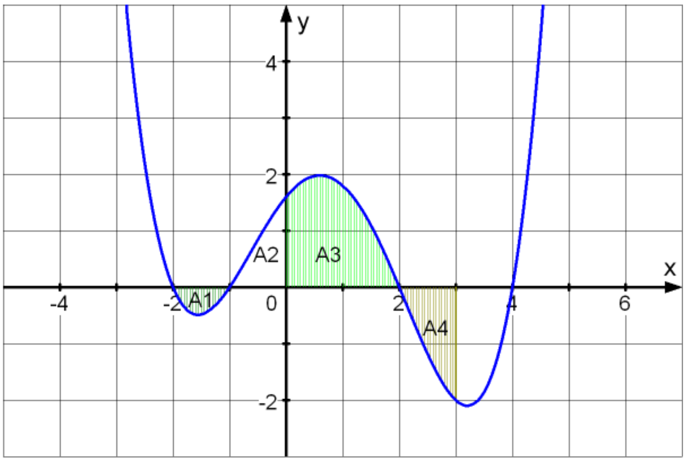
3 Flächenbilanzen



Gegeben : $A_1 = 0{,}3$, $A_2 = 0{,}8$, $A_3 = 2{,}9$ und $A_4 = 1{,}1$

a) $\int_{-2}^{0} f(x)dx = -0{,}3$

b) $\int_{-2}^{3} f(x)dx = -0{,}3 + 0{,}8 + 2{,}9 - 1{,}1 = 2{,}3$

c) $\int_{0}^{3} f(x)dx = 2{,}9 - 1{,}1 = 1{,}8$

d) $\int_{-2}^{0} f(x)dx = -0{,}3$

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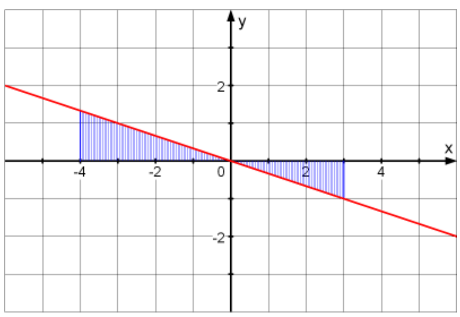
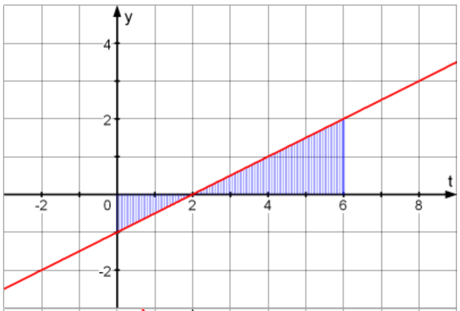
4 Flächenbilanz

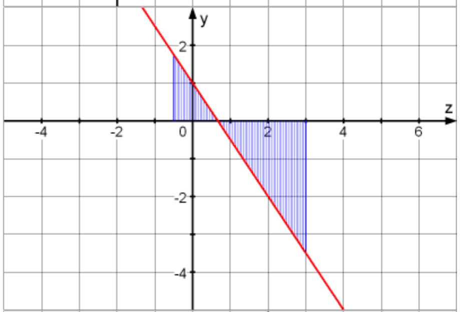
a) $\int_{-4}^{3} (-\frac{1}{3}x)dx = \frac{1}{2}\cdot 4\cdot\frac{4}{3} - \frac{1}{2}\cdot 3\cdot 1 = \frac{7}{6}$



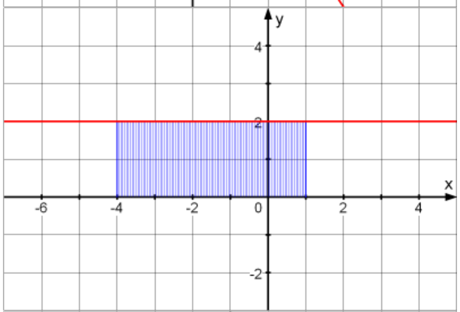
b) $\int_0^6 (\frac{1}{2}t - 1)dt = -\frac{1}{2}\cdot 2\cdot 1 + \frac{1}{2}\cdot 4\cdot 2 = 3$



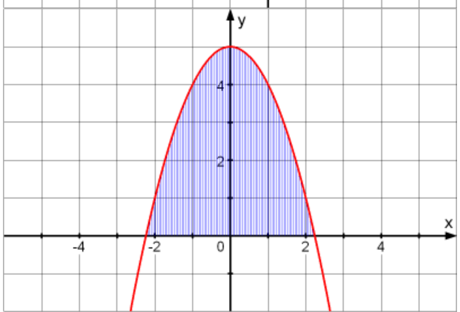
c) $\int_{-0,5}^{3} (-1,5z + 1)dz = \frac{1}{2}\cdot\frac{7}{4}\cdot\frac{7}{6} - \frac{1}{2}\cdot\frac{7}{3}\cdot\frac{7}{2} = -3\frac{1}{8}$



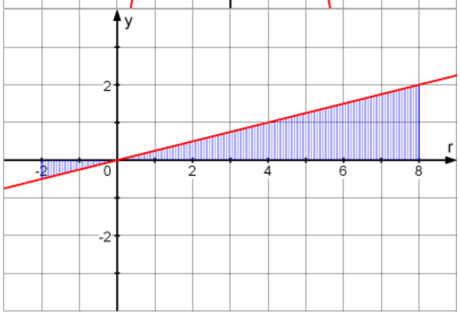
d) $\int_{1}^{-4} 2dx = -10$



e) $\int_{-\sqrt{5}}^{\sqrt{5}} (-x^2 + 5)dx = \frac{20}{3}\sqrt{5}$



f) $\int_{8}^{-2} \frac{1}{4}rdr = -\frac{1}{2}\cdot 8\cdot 2 + \frac{1}{2}\cdot 2\cdot\frac{1}{2} = -7,5$



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5 Vozeichen

a) $\int_{10}^{80} x^2 dx > 0$

b) $\int_{-10}^{11} -x^4 dx < 0$

c) $\int_{0}^{2\pi} \sin t\,dt = 0$

d) $\int_{-4}^{2} u^3 du < 0$ e) $\int_{1}^{-2} (-x^4+4)dx < 0$ f) $\int_{3}^{7} -\frac{1}{4}(s-5)^2 ds < 0$

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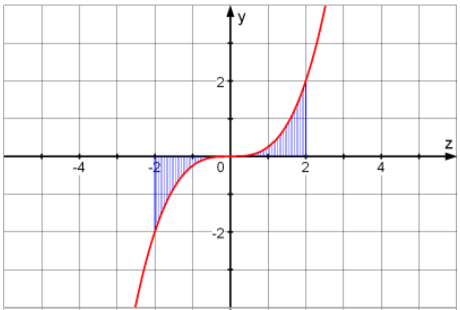
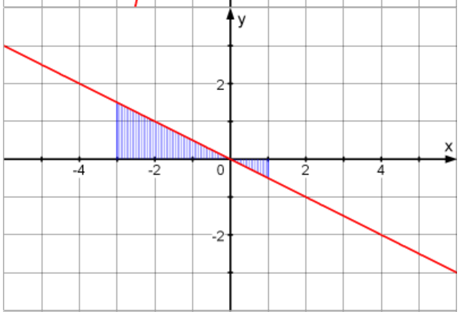
5 Vorgegebene Integralwerte

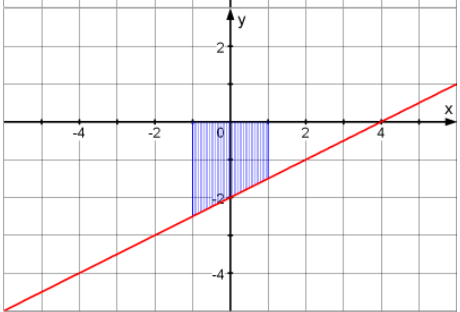
a) $\int_{-2}^{2} f(z)dz = 0$



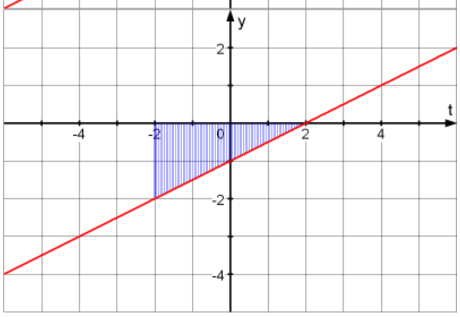
b) $\int_{-3}^{1} f(x)dx = 2$



c) $\int_{1}^{-1} f(x)dx = 4$



d) $\int_{-2}^{2} f(t)dt = -4$



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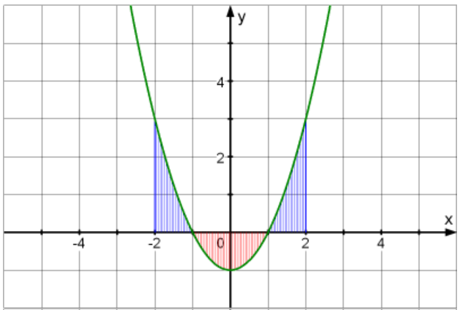
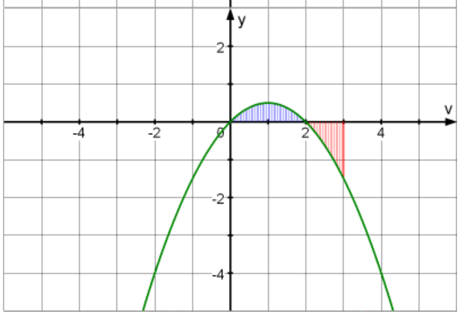
7 Flächenbilanz

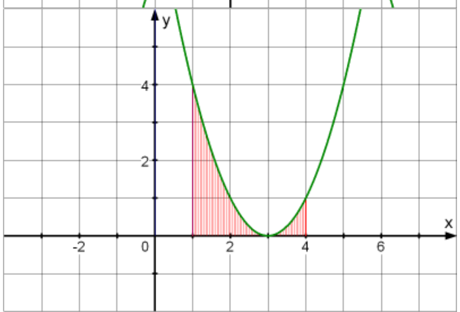
a) $\int_{-2}^{2}(x^2-1)dx = \frac{4}{3}$



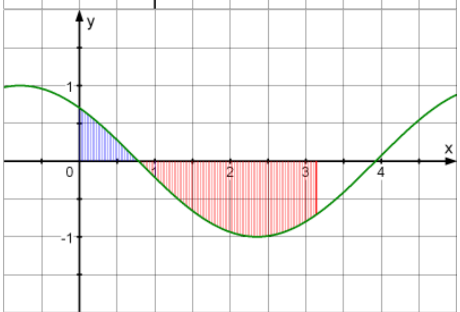
b) $\int_{0}^{3} -0{,}5v\cdot(v-2)dv = 0$



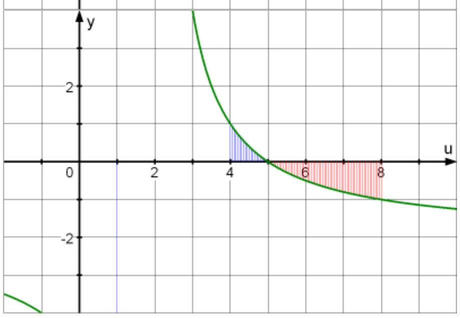
c) $\int_{0}^{\pi}\cos(x+\frac{\pi}{4})dx = -\sqrt{2}$



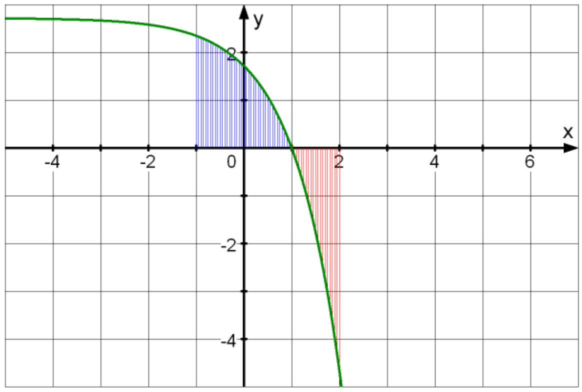
d) $\int_{4}^{1}(x-3)^3dx = \frac{15}{4}$



e) $\int_{4}^{6}(\frac{6}{u-2}-2)du = 6\cdot\ln3 - 8 \approx -1{,}408$



f) $\int_1^2 (-e^x + e)dx = -e^2 + 3e - \frac{1}{e}$



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8 Integral und Fläche

	f(x)	a	b	$\int_a^b f(x)dx$	A
a)	$f(x) = 0{,}5\cdot(x+1)(x-2)$	0	3	0,75	2,25
b)	$f(x) = \frac{1}{4}x\cdot(x+1{,}5)\cdot(x-2)\cdot(x-3)$	$-1{,}5$	3	$-\frac{720}{2560}$	$\frac{8921}{2560}$
c)	$f(x) = 0{,}5\cdot(x^2-1)(x-2{,}5)$	1	1,75	$\frac{8107}{6144}$	$\frac{12373}{6144}$
d)	$f(x) = 2\cdot\sin(4x)$	$-\frac{\pi}{2}$	$\frac{\pi}{2}$	1	7

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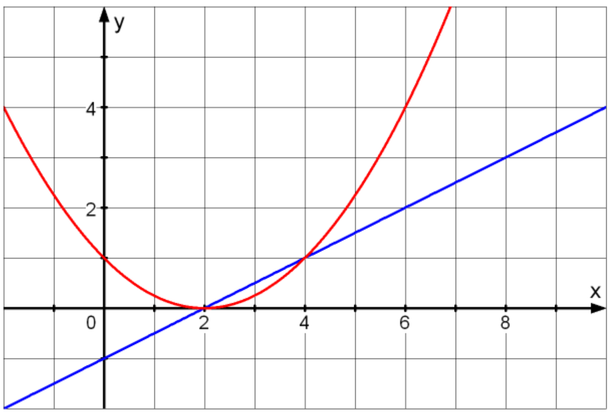
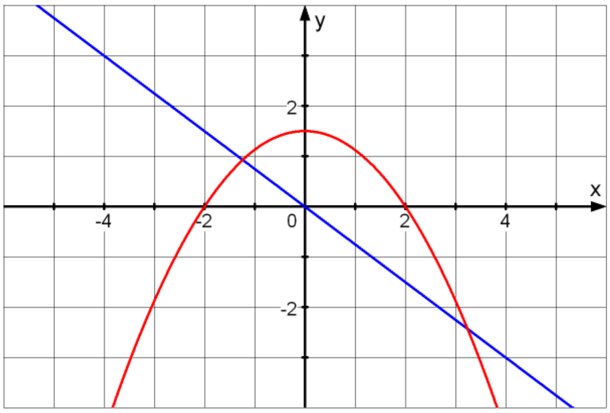
9 Integralfunktionen

a)



b)



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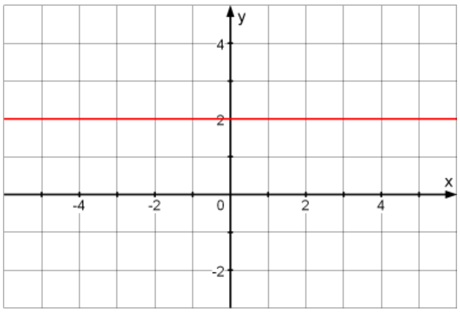
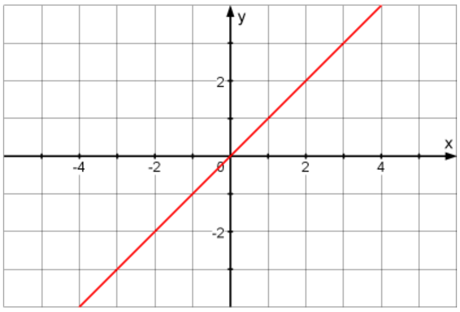
10 Elementare Bestimmung von Integralfunktionen

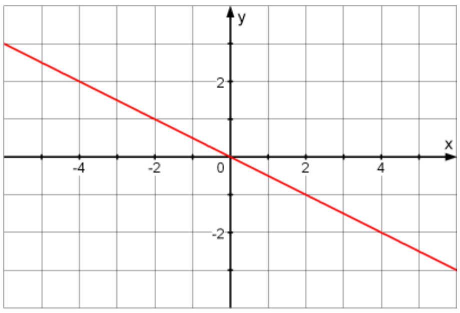
a) $I_0(x) = \int_0^x f(t)dt = 2x$



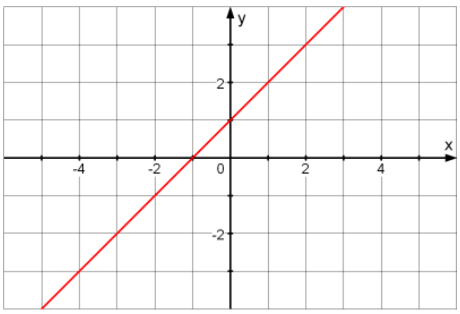
b) $I_0(x) = \int_0^x f(t)dt = \frac{1}{2}x \cdot x = \frac{1}{2}x^2$



c) $I_0(x) = \int_0^x f(t)dt = -\frac{1}{2}x \cdot 0{,}5x = -\frac{1}{4}x^2$



a) $I_0(x) = \int_0^x f(t)dt = x \cdot 1 + \frac{1}{2}x \cdot x = = \frac{1}{2}x^2 + x$



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11 Zuordnung

Der Graph von $I_0 = \int_0^x f(t)dt$

muss die Nullstelle $x_0 = 0$ besitzen und zwei Extrema bei $x = 0$ und $x = 3$ besitzen $\rightarrow$ 4.

Der Graph von $I_1 = \int_1^x h(t)dt$

muss die Nullstelle $x_0 = 0$ besitzen und drei Extrema bei $x = -1$ und $x = 1$ sowie $x = 3$ besitzen $\rightarrow$ 5.

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12 Zunahme oder Abnahme

$\frac{1}{2}\cdot 2{,}5\cdot 1{,}5 - \frac{1}{2}\cdot(2{,}5+1{,}5)\cdot 1 < 0 \rightarrow$ Abnahme

$\int_{0,5}^{2,5}[4\cdot(x-1{,}5)^2-1]dx = \frac{2}{3} > 0 \rightarrow$ Zunahme

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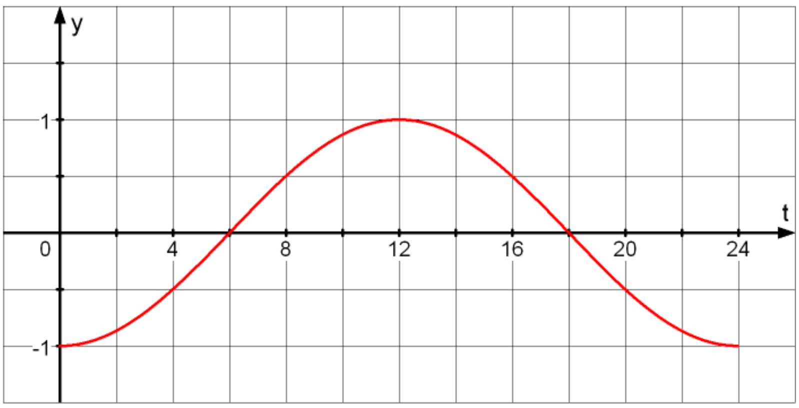
14 Lufttemperatur



a) 0.00 - 6.00 Uhr: Abnahme der Lufttemperatur, da negative Änderungsrate

6.00 - 18.00 Uh: Zunahme der Lufttemperatur, da positive Änderungsrate

18.00 - 24.00: Abnahme der Lufttemperatur, da negative Änderungsrate

b) 0.00 und 24.00 Uhr: stärkste Abnahme

12.00 Uhr särkste Zunahme

c) Tiefste Temperatur: 6.00 Uhr

Höchste Temperatur: 18.00 Uhr

d) $\int_{6}^{18} \cos[\frac{2\pi}{24} \cdot (t-12)]dt \approx 7{,}6 \left(K\right)$

G 15

$A = \frac{1}{2} \cdot \frac{5\pi}{4} \cdot (15\text{ m})^2 \approx 441{,}786\text{ m}^2 = 0{,}0441786\text{ ha} = 44178{,}6\text{ dm}^2$

G 16

a) $y = x+2 \wedge y = 2x+1 \quad \Rightarrow \quad x = 1$ und $y = 3$

b) $x+y = 4 \wedge 2x-y = 2 \quad \Rightarrow \quad x = 2$ und $y = 2$

c) $2x-y = 2 \wedge x-3y = 5 \quad \Rightarrow \quad x = 0{,}2$ und $y = -1{,}6$

d) $\frac{1}{2}x+2y = -\frac{11}{4} \wedge -\frac{5}{4}x+\frac{1}{2}y = 0 \quad \Rightarrow \quad x = -0{,}5$ und $y = -1{,}25$
