Das Integral

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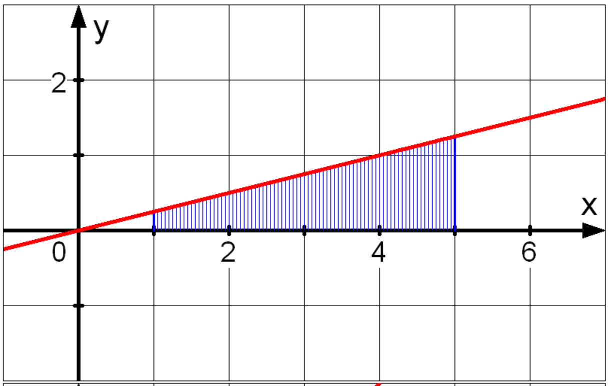
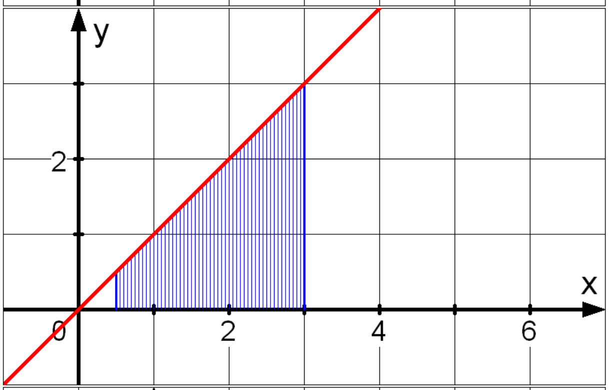
2 Bestimmte Integrale

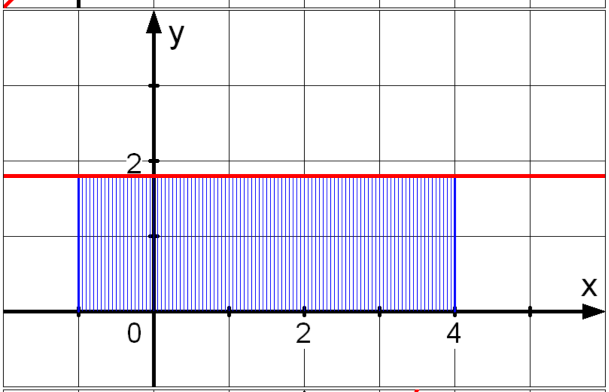
$$\int_{1}^{5} 0{,}25x\,dx = \frac{1}{2}\cdot\left(0{,}25+1{,}25\right)\cdot 4 = 3$$



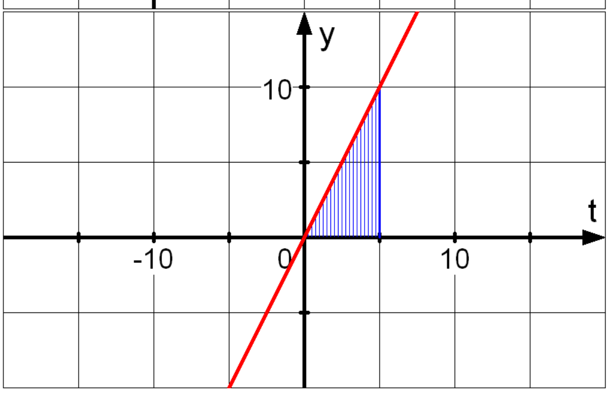
$$\int_{0{,}5}^{3} x\,dx = \frac{1}{2}\cdot\left(0{,}5+3\right)\cdot 2 = 3{,}5$$



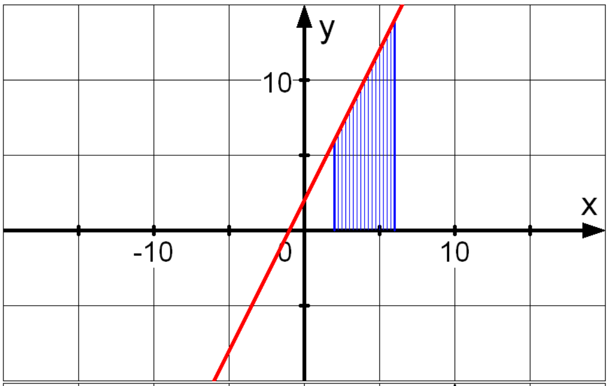
$$\int_{-1}^{4} 1{,}8\,dx = 1{,}8\cdot 5 = 9$$



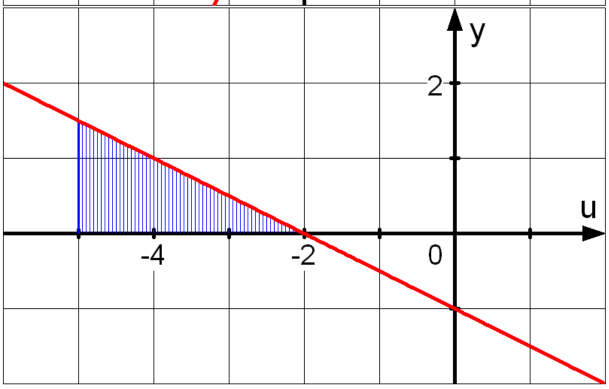
$$\int_{0}^{5} 2t\,dx = \frac{1}{2}\cdot 5\cdot 10 = 25$$



$$\int_{6}^{2}(2x+2)dx = -\frac{1}{2}\cdot\left(6+14\right)\cdot 4 = -20$$

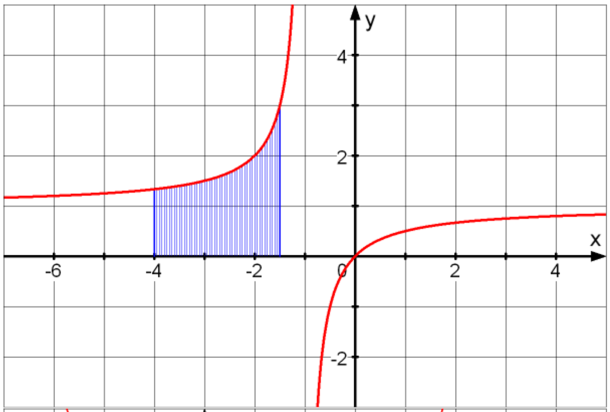


$$\int_{-5}^{-2}(-0{,}5u-1)du = \frac{1}{2}\cdot 1{,}5\cdot 3 = 2{,}25$$

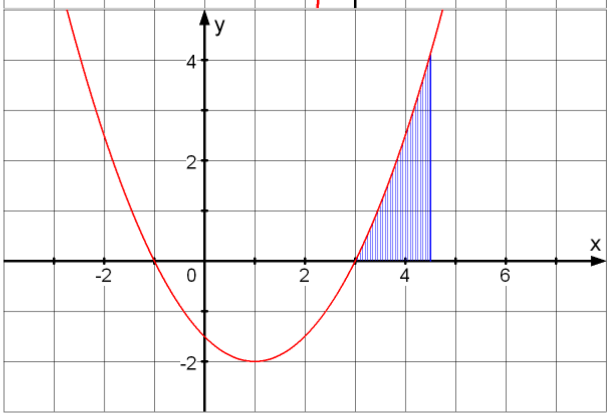


3

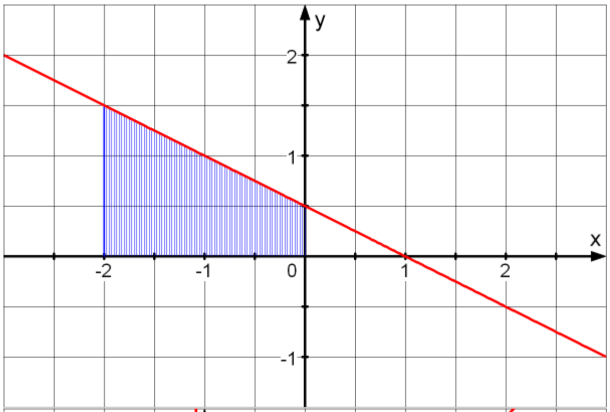
$$\int_{-4}^{-1{,}5}\left(\frac{-1}{x+1}+1\right)dx$$



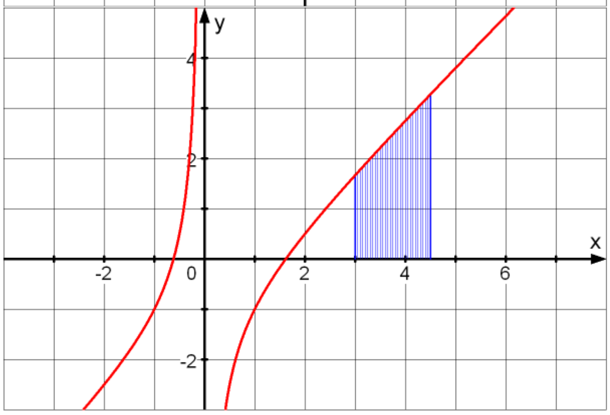
$$\int_{3}^{4{,}5}[0{,}5\cdot(x-1)^2-2)dx$$



$$\int_{-2}^{0}(-0{,}5x+0{,}5)\,dx$$



$$\int_{3}^{4{,}5}\frac{x^2-x-1}{x}\,dx$$



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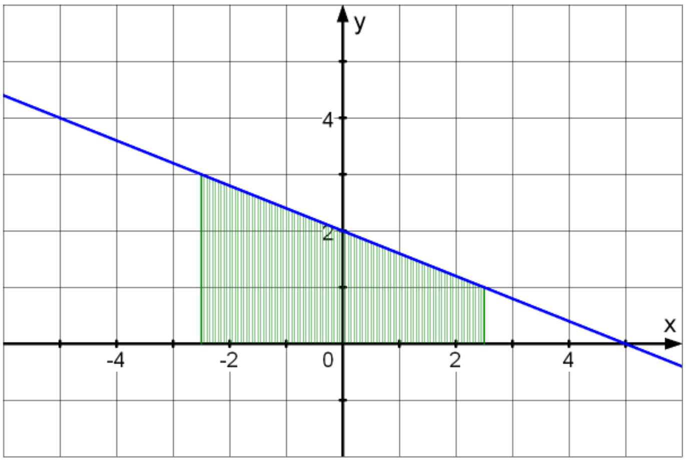
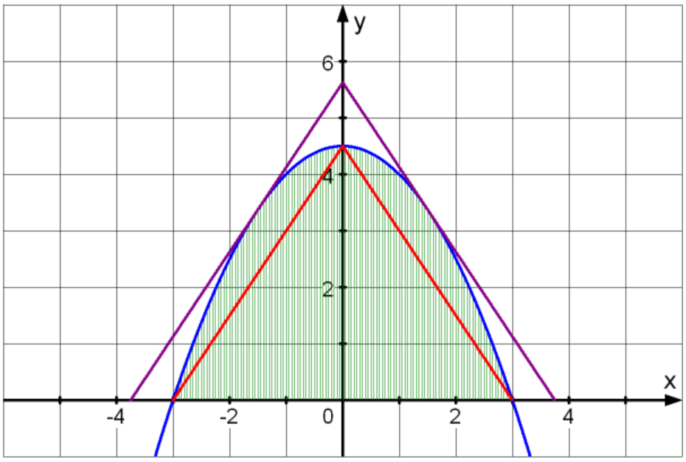
4 Rechnung und Abschätzung

a)



$$A = \frac{1}{2}\cdot(3+1)\cdot 5 = 10$$

b)



$\frac{1}{2} \cdot 6 \cdot 4{,}5 < A < \frac{1}{2} \cdot 7{,}5 \cdot 6, \quad \Leftrightarrow \quad 13{,}5 < A < 24{,}75$

Abschätzung durch Mittelwert: $A \approx \frac{13{,}5 + 24{,}75}{2} = 19{,}125$

Exakter Wert: $A = 18$

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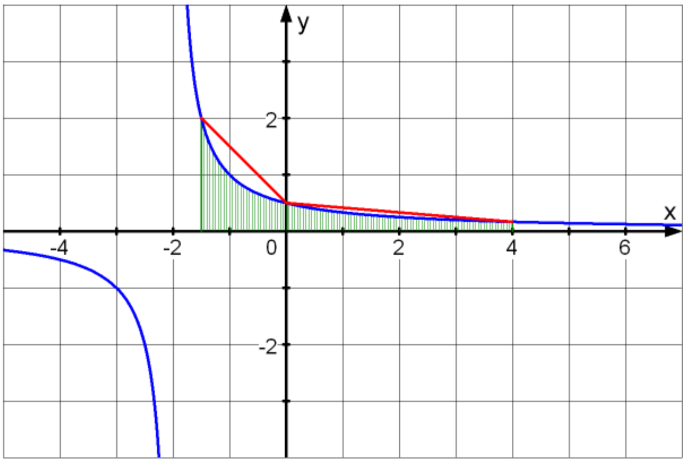
c)



$A \approx \frac{1}{2} \cdot \left(2 + \frac{1}{2}\right) \cdot \frac{3}{2} + \frac{1}{2} \cdot \left(\frac{1}{2} + \frac{1}{6}\right) \cdot 4 = 3\frac{5}{24}$

Exakter Wert: $A = \ln 12 \approx 2{,}48$

Zerlegung in drei Trapeze ergibt:

$A \approx \frac{1}{2} \cdot (2 + 1) \cdot \frac{1}{2} + \frac{1}{2} \cdot \left(1 + \frac{1}{2}\right) \cdot 1 + \frac{1}{2} \cdot \left(\frac{1}{2} + \frac{1}{6}\right) \cdot 4 = \frac{17}{6} \approx 2{,}83$

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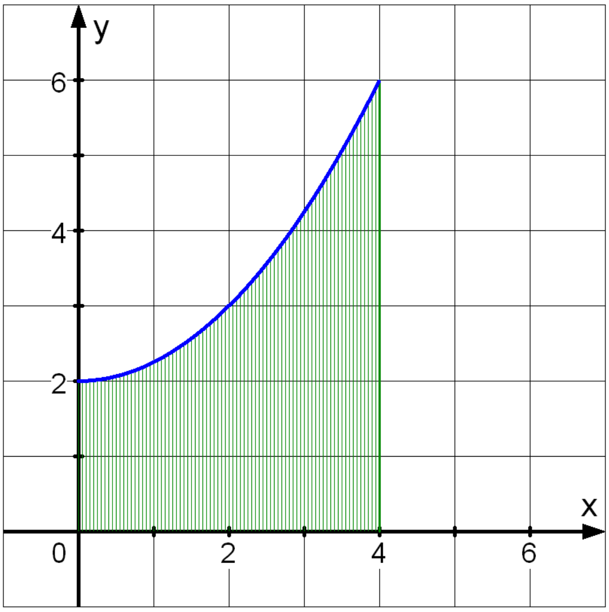
5 Unter- und Obersummen

a)



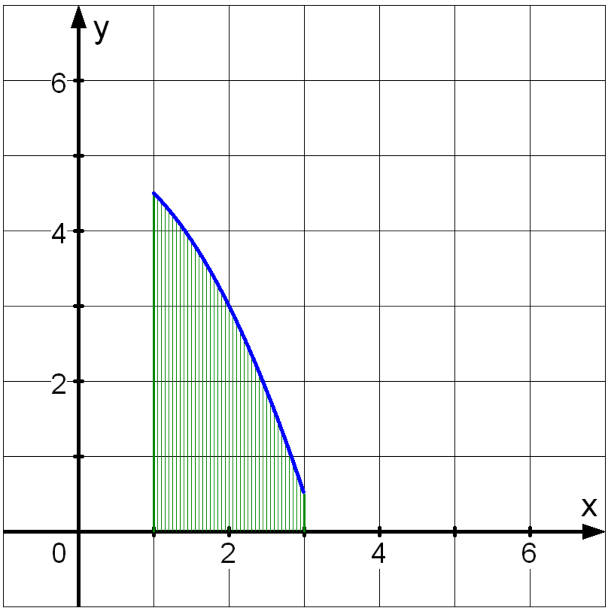
	0	0,5	1	1,5	2	2,5	3	3,5
$f(x) = 0{,}25x^2 + 2$	2	2,0625	2,25	2,5625	3	3,5625	4,25	5,0625

$U_8 = 24{,}75 \cdot 0{,}5 = 12{,}375$

	0,5	1	1,5	2	2,5	3	3,5	4
$f(x) = 0{,}25x^2 + 2$	2,0625	2,25	2,5625	3	3,5625	4,25	5,0625	6

$o_8 = 28{,}75 \cdot 0{,}5 = 14{,}275$

b)



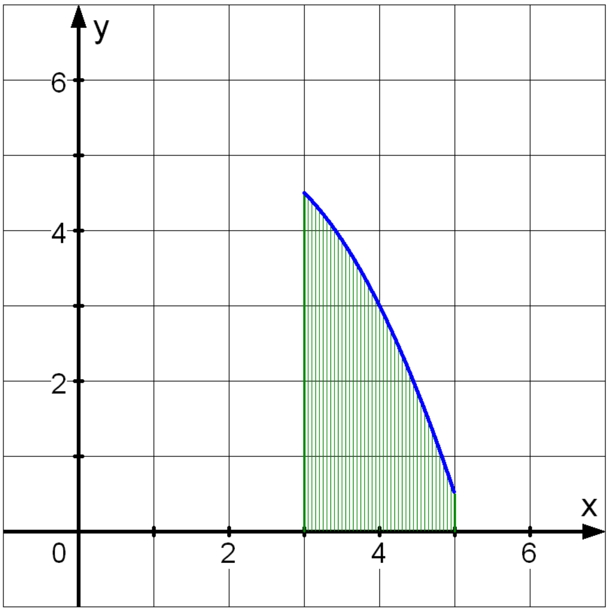
	1,25	1,5	1,75	2	2,25	2,5	2,75	3
$f(x) = -0{,}5x^2 + 5$	4,21875	3,875	3,46875	3	2,46875	1,875	1,21875	0,5

$U_8 = 20{,}625 \cdot 0{,}25 = 5{,}15625$

	1	1,25	1,5	1,75	2	2,25	2,5	2,75
$f(x) = -0{,}5x^2+5$	4,5	4,21875	3,875	3,46875	3	2,46875	1,875	1,21875

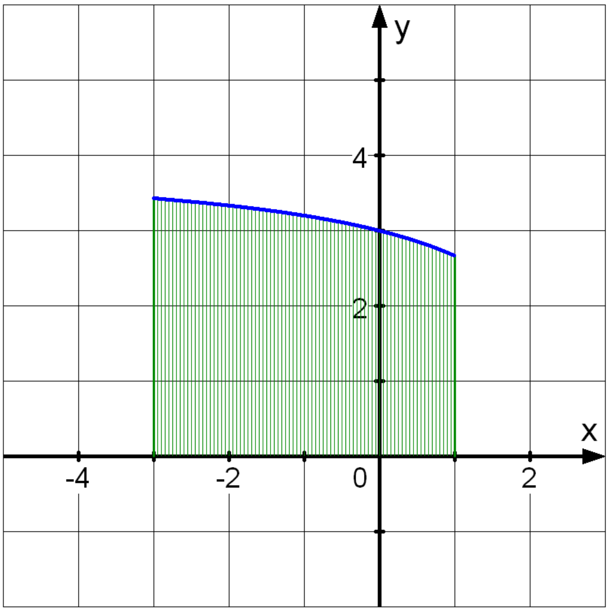
$O_8 = 24{,}625 \cdot 0{,}25 = 6{,}15625$

c)



$U_8 = 20{,}625 \cdot 0{,}25 = 5{,}15625$ und $U_8 = 20{,}625 \cdot 0{,}25 = 5{,}15625$

d)



	−2,5	−2	−1,5	−1	−0,5	0	0,5	1
$f(x) = \frac{4}{x-4}+4$	$\frac{44}{13}$	$\frac{10}{3}$	$\frac{36}{11}$	$\frac{16}{5}$	$\frac{28}{9}$	3	$\frac{20}{7}$	$\frac{8}{3}$

$U_8 \approx 12{,}41$

	-3	$-2,5$	-2	$-1,5$	-1	$-0,5$	0	$0,5$
$f(x) = \frac{4}{x-4} + 4$	$\frac{24}{7}$	$\frac{44}{13}$	$\frac{10}{3}$	$\frac{36}{11}$	$\frac{16}{5}$	$\frac{28}{9}$	3	$\frac{20}{7}$

$O_8 \approx 14{,}39$

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6 Noramalparabel

a) $U_n = \frac{2}{6} \cdot \left(2 - \frac{1}{n}\right) \cdot \left(1 - \frac{1}{n}\right)$ und damit $U_{10} = \frac{2}{6} \cdot \left(1 - \frac{1}{10}\right) \cdot \left(2 - \frac{1}{10}\right) = 0{,}57$

b) $\lim\limits_{n \to \infty} U_n = \frac{4}{3}$

---

7 CAS

a) $\int\limits_{-\sqrt{5}}^{\sqrt{5}} (-0{,}5x^2 + 2{,}5)dx \approx 7{,}45$

b) $\int\limits_{-1-\sqrt{5}}^{0} (0{,}25x^3 + 0{,}5x^2 - x)dx \approx 4{,}03$

c) $\int\limits_{-\frac{2}{3}}^{4} \left(\frac{-1}{x+2} + 0{,}75\right)dx \approx 2{,}00$

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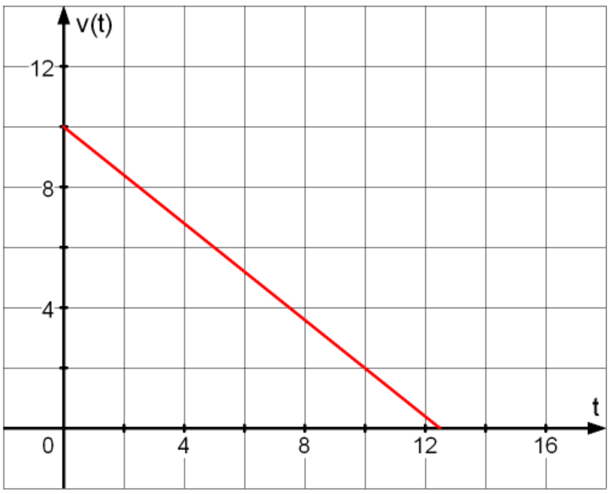
8 Zeit-Geschwindigkeits-Funktion



a) $v(t) = 0 \quad \Leftrightarrow \quad -0{,}8t + 10 = 0 \quad \Leftrightarrow \quad t = 12{,}5$

b) $\int_{0}^{12,5} v(t)dt = \frac{1}{2} \cdot 10 \cdot 12,5 = 62,5 \ \ (m)$

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9 Schadstoffausstoß

a)

b) $S = \int_{0}^{24} [5 \cdot \sin(0,25t) + 10]dt$

c) $S \approx 10 \ \frac{g}{h} \cdot 24 \ h = 240 \ g$

d) $S_{CAS} \approx 240,8$

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10 Halbkreis

a) $\overline{OP}^2 = x^2 + \sqrt{1-x^2}^{\,2} = x^2 + 1 - x^2 = 1 \quad \Rightarrow \quad \overline{OP} = 1$

b) $A = \int_{-1}^{1} \sqrt{1-x^2}dx = 1,570...$

c) $A = \frac{\pi}{2}$

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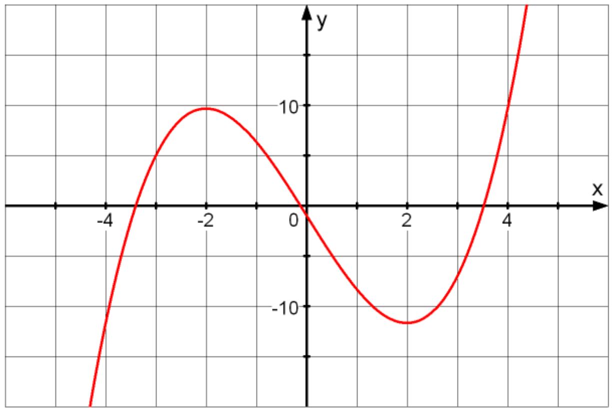
11 Extremstellen

a) $f(x) = \frac{2}{3}x^3 - 8 \cdot x - 1 \quad \Rightarrow \quad f'(x) = 2x^2 - 8 = 0 \quad \Leftrightarrow \quad x = -2 \ \vee \ x = 2$

	$-\infty < x < -2$	$-2 < x < 2$	$2 < x < \infty$
$f'(x)$	$+$	$-$	$+$

$H\left(-2 \mid 9\frac{2}{3}\right)$ ist ein Hochpunkt und $T\left(2 \mid -9\frac{2}{3}\right)$ ist ein Tiefpunkt.



b) $f(x) = \frac{1}{8}x^4 - \frac{9}{4}x^2 + 4 \quad \Rightarrow \quad f'(x() = \frac{1}{2}x^3 - \frac{9}{2}x = \frac{1}{2}x \cdot (x-3) \cdot (x+3) = 0$

$\Leftrightarrow \quad x' = -3 \vee x = 0 \vee x = 3$

	$-\infty < x < -3$	$-3 < x < 0$	$0 < x < 3$	$3 < x < \infty$
x	$-$	$-$	$+$	$+$
$x+3$	$-$	$+$	$+$	$+$
$x-3$	$-$	$-$	$-$	$+$
$f'(x)$	$-$	$+$	$-$	$+$

$T_1\left(-3 \mid -6\frac{1}{8}\right)$ und $T_2\left(3 \mid -6\frac{1}{8}\right)$ sind Tiefpunkte und $H\left(0 \mid 4\right)$ ist ein Hochpunkt.