2 Verhalten im Unendlichen
===
2 Asymptoten

	Funktion	**senkrechte Asymptoten**	**waggrecht bzw. schräge Asymptoen**
a)	$f(x) = \frac{4}{2x+1}$	$x = -\frac{1}{2}$	$y = 0$
b)	$f(x) = \frac{x-1}{x+1}$	$x = -1$	$y = 1$
c)	$f(x) = \frac{3-2x}{4x^2-1}$	$x = -\frac{1}{2}$ und $x = \frac{1}{2}$	$y = 0$
d)	$f(x) = \frac{3+2x^2}{4x^2+1}$	---	$y = \frac{1}{2}$
e)	$f(x) = \frac{2x^3-1}{4x^2-x^3}$	$x = 4$ und $x = 0$	$y = 2$
f)	$f(x) = \frac{3x^2-2x}{2x-1}$	$x = \frac{1}{2}$	$y = \frac{3}{2}x - \frac{1}{4}$
g)	$f(x) = x - \frac{1}{x-1}$	$x = 1$	$y = x$
h)	$f(x) = \frac{3x}{(x-2)^2}$	$x = 2$	$y = 0$

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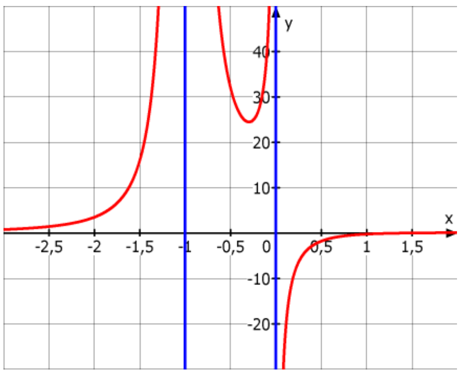
3 Beschreibung von Graphen

a) $f(x) = \frac{2x-3}{x\cdot(1+x)^2}$



Waagrechte Asymptote: $y = 0$

Senkrechte Asymptoten: $x = -1$ (Pol gerader Ordnung) und $x = 0$ (Pol ungerader Ordnung)

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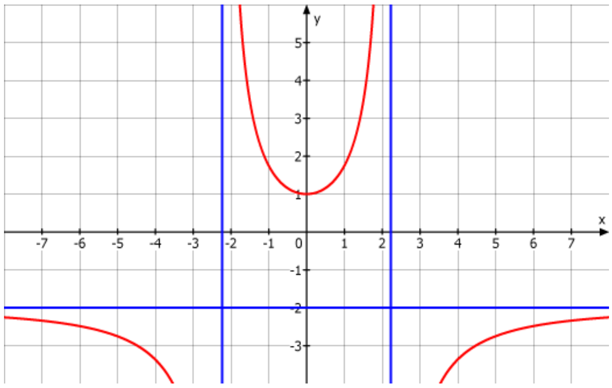
b) $f(x) = \frac{2x^2+5}{5-x^2}$



Waagrechte Asymptote: $y = -2$

Senkrechte Asymptoten: $x = -\sqrt{5}$ und $x = \sqrt{5}$ (Pole ungerader Ordnung)

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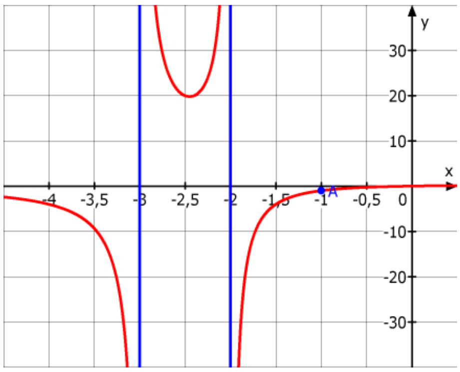
c) $f(x) = \dfrac{2x^2 + 2x}{(x+1)(x+2)(x+3)} = \dfrac{2x}{(x+2)\cdot(x+3)}$



Waagrechte Asymptote: $y = 0$

Senkrechte Asymptoten: $x = -3$ und $x = -2$ (Pole ungerader Ordnung)

$$\lim_{x \to -1} \frac{2x^2 + 2x}{(x+1)\cdot(x+2)\cdot(x+3)} = \lim_{x \to -1} \frac{2x}{(x+2)\cdot(x+3)} = -1$$

---

4 Asymptoten → Funktion

$f_1 : x \to \dfrac{x+1}{2-x}$ $\qquad$ $f_2 : x \to \dfrac{0{,}5x^2 - 1}{x+2}$ $\qquad$ $f_3 : x \to \dfrac{3x+2}{2x-4}$

$f_4 : x \to \dfrac{1+2x^2}{(x-2)^2}$ $\qquad$ $f_5 : x \to \dfrac{5-x^2}{4-2x}$ $\qquad$ $f_6 : x \to \dfrac{0{,}5x-2}{x-2}$

Asymptoten : $y = \dfrac{1}{2}x + 1$ und $x = 2$

Nur der Graph f_5 hat die angegebenen Asymptoten.

Grund :

Die Definitionslücke muss 2 und der Grad des Zählerpolynoms muss um 1 größer als der des Polynoms im Nenner sein

---

5 Äquivalente Terme

1 $\left| \frac{x+1}{x-1} \right.$ = U $\left| \frac{x}{x-1} - \frac{1}{1-x} \right.$ 2 $\left| 2x+1-\frac{2}{x} \right.$ = D $\left| \frac{2x^2+x-2}{x} \right.$

3 $\left| (x-2)^2 \right.$ = A $\left| x^2-4x+4 \right.$ 4 $\left| \frac{3x-2x^2}{4x^2+1} \right.$ = K $\left| \frac{\frac{3}{x}-2}{4+\frac{1}{x^2}} \right.$

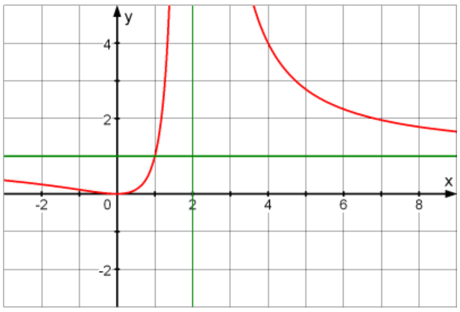
5 $\left| \frac{-x^2-x+5}{2-x} \right.$ = A $\left| x+3-\frac{1}{x-2} \right.$ 6 $\left| 2x^2-12x+16 \right.$ = K $\left| 2\cdot(x-2)\cdot(x-4) \right.$

"KAKADU"

---

6 Schnittpunkte mit den Asymptoten

$$\frac{x^2}{(x-2)^2} = 1 \quad \Rightarrow \quad 0 = -4x+4 \quad \Leftrightarrow \quad x = 1 \quad S\left(1 \middle| 1\right)$$



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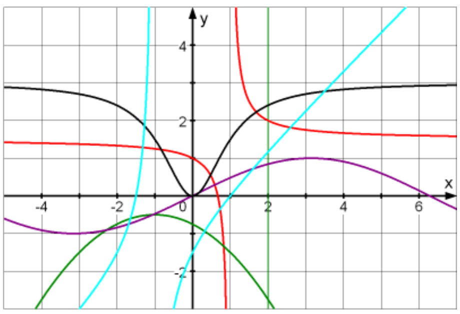
8 Zuordnung von Graphen zu Funktionsgleichungen



Rot: $f(x) = \frac{3x-2}{3x-2}$

Violett : $p(x) = \sin\left(\frac{1}{2}x\right)$

Schwarz : $m(x) = \frac{3x^2}{x^2+1}$

Grün : $n(x) = -\frac{1}{4}x^2 - \frac{1}{2}x - \frac{3}{4}$

Himmelblau : $g(x)\ x - \frac{1}{2} - \frac{1}{x+1}$

---

9 Vorgegebene Asymptoten

a) $y = -1$ und $x = 0$: $f(x) = \frac{1-x}{x}$

b) $x = 1{,}5$: $f(x) = \frac{1}{2x-3}$

c) $x = -1$ und $y = x-2$: $f(x) = x - 2 + \frac{1}{x+1}$

d) $x = -2$ und $x = 2$: $f(x) = \frac{1}{x^2-4}$

e) $y = x$: $y = x + \frac{1}{x}$

f) $y = 0$ und $x = \sqrt{2}$: $f(x) = \frac{1}{x^2-2}$

g) Der Graph einer gebrochen rationalen Funktion kann höchstens eine Asymptote im Unendlichen haben.

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10 Sauerstoffgehalt in einem Teich

$f(t) = \frac{at^2+bt+c}{t^2+1}$

t (in Tagen)	$f(t)\left(\text{in } \frac{mg}{l}\right)$

0	**12,0**
1	**4,5**
2	**6,0**
3	**7,5**

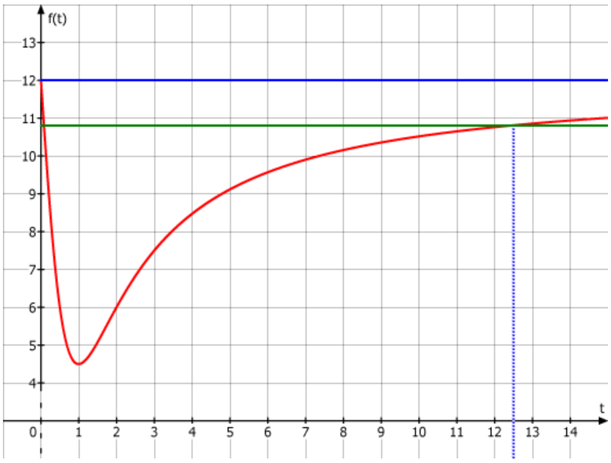
(1) $c = 12$

(2) $a + b + 12 = 9$

(3) $4a + 2b + 12 = 30$

ergibt $a = 12$ und $b = -15$

b)



c) $\frac{12t^2 - 15t + 12}{t^2 + 1} = 10{,}8 \quad \Leftrightarrow \quad 1{,}2t^2 - 15t + 1{,}2 = 0 \quad \Leftrightarrow \quad t^2 - 12{,}5t + 1 = 0$

$$\Rightarrow \quad t = \frac{12{,}5 + \sqrt{152{,}25}}{2} \approx 12{,}4$$

---

11 Spezialfall der Polynomdivision

a) $f(x) = \frac{4x - 3}{0{,}5x + 2} = \frac{4x + 16 - 19}{0{,}5x + 2} = 8 - \frac{19}{0{,}5x + 2}$

Asymptote: $y = 8$

b) $f(x) = \frac{3x^2 - 1}{3 - x - 2x^2} = \frac{3x^2 + 1{,}5x - 4{,}5 - 1{,}5x + 3{,}5}{-2x^2 - x + 3} = -1{,}5 - \frac{1{,}5x - 3{,}5}{3 - x - 2x^2}$

Asymptote: $y = -1{,}5$

c) $g(x) = \frac{0{,}5x^2 + 2}{2x + 1} = \frac{0{,}5x^2 + 0{,}25 + 1{,}75}{2x + 1} = \frac{1}{4} + \frac{7}{8x + 4}$

Asymptote: $y = \frac{1}{4}$

d) $g(x) = \frac{2-5x^2}{2x^2-5} = \frac{-5x^2+12{,}5-10{,}5}{2x^2-5} = -2{,}5 - \frac{21}{4x^2-10}$

Asymptote: $y = -2{,}5$

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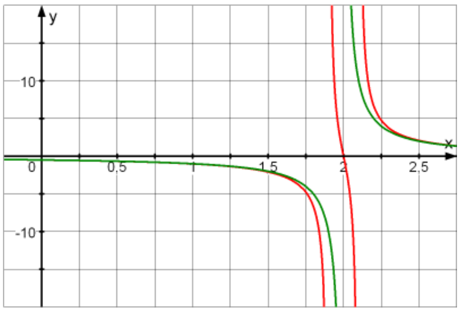
12 "Fast kürzbar"



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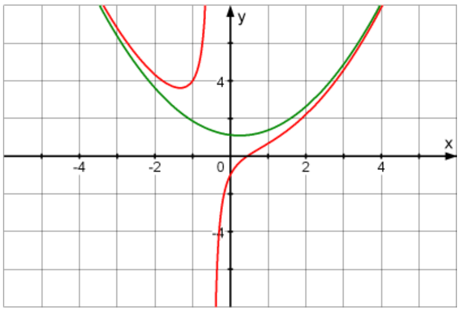
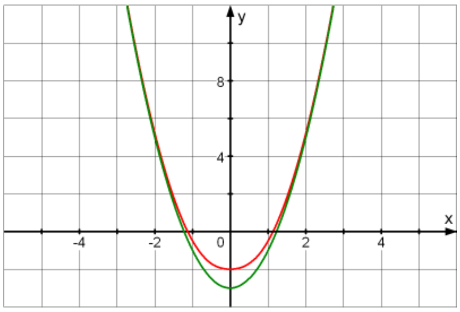
13 Asymptotische Kurven

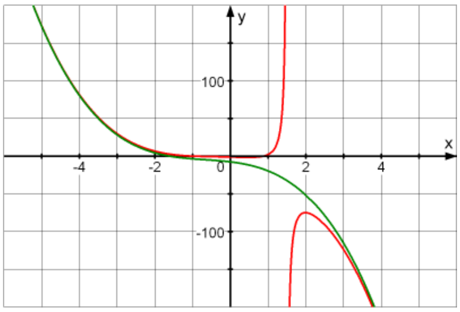
a) $f(x) = \frac{x^3+2x-1}{2x+1} = \frac{1}{2}x^2 - \frac{1}{4}x + \frac{9}{8} - \frac{17}{16x+8} \quad \left(\frac{17}{168} \text{ bzw. } \frac{17}{1608}\right)$



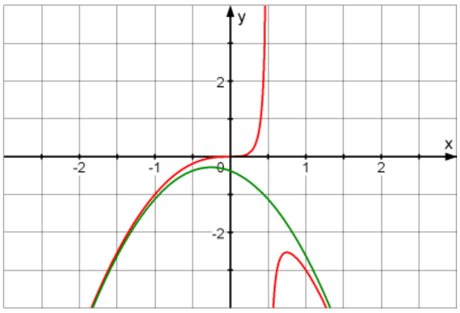
b) $f(x) = \frac{2x^4-x^2-2}{x^2+1} = 2x^2 - 3 + \frac{1}{x^2+1} \quad \left(\frac{1}{101}; \frac{1}{10001}\right)$



c) $f(x) = \frac{-4x^4 - 2x^3 + 3x}{2x-3} = -2x^3 - 4x^2 - 6x - 7{,}5 - \frac{45}{4x-6} \quad \left(\frac{45}{34} \text{bzw. } \frac{45}{394}\right)$



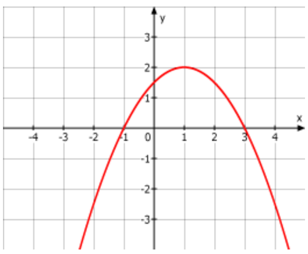
d) $f(x) = \frac{3x^3}{1-2x} = -\frac{3}{2}x^2 - \frac{3}{4}x - \frac{3}{8} + \frac{3}{8-16x} \quad \left(\frac{3}{152} \text{ bzw. } \frac{3}{1592}\right)$



G 14 Parabeln

a) $p_a(x) = a \cdot (x+1) \cdot (x-3)$

b) $2 = a \cdot (1+1) \cdot (1-3) \quad \Rightarrow \quad a = -\frac{1}{2}$



c) Die Scheitel der Schargraphen liegen auf der Geraden $x = 1$.

$(1 \mid 0)$ kann kein Scheitelpunkt sein, da die Nullstellen jeder Scharfunktion -1 und 3 sind.

G 15 Anzahl der Lösungen linearer Gleichungssysteme

a) $\begin{aligned} 0{,}3r - s &= 1{,}2 \\ 0{,}4r + 5s &= 5{,}4 \end{aligned} \quad \Rightarrow \quad r = 6 \;\wedge\; s = 0{,}6$

b $\begin{aligned} 0{,}4A - B &= 1{,}5 \\ 2A - 5B &= 2 \end{aligned} \quad \Rightarrow$ Das Gleichungssystem besitzt keine Lösung.

c) $\begin{aligned} -1{,}5\alpha - 3\beta &= -4{,}5 \\ \alpha + 2\beta &= 3 \end{aligned} \quad \Rightarrow \quad L = \left\{ \left(3 - 2\beta \mid \beta\right) \,\middle|\, \beta \in \mathbb{R} \right\}$

d) $\begin{aligned} \gamma + 3\varepsilon &= \frac{1}{3} \\ 3\gamma - \frac{1}{3}\varepsilon &= 1 \end{aligned} \quad \Rightarrow \quad \gamma = \frac{1}{3} \;\wedge\; \varepsilon = 0$

e) $\begin{aligned} 6\cdot(3x - 2y) + 5\cdot(x + 2y) &= 21 \\ 7\cdot(4x + y) - 4\cdot(2x - 5y) &= 47 \end{aligned} \quad \Rightarrow \quad x = y = 1$

f) $\begin{aligned} \frac{1}{4}\cdot(2x - 1) - \frac{1}{3}\cdot(+xy) &= -\frac{1}{4} \\ \frac{1}{2}\cdot(y - 1) + \frac{1}{5}\cdot(y - x) &= 4 \end{aligned} \quad \Rightarrow \quad x = 30 \;\wedge\; y = 15$
