

## Kettenregel

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1. Bestimme  $f'(x)$ .

a)  $f(x) = (2x + 1)^3$

b)  $f(x) = 2 \cdot (x + 1)^3$

c)  $f(x) = 2 + (x + 1)^3$

d)  $f(x) = \sqrt{\frac{1}{2}x + 1}$

e)  $f(x) = a \cdot \cos(bx + c)$

f)  $f(x) = 3 \cdot e^{1-2x}$

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2. Bestimme  $f'(x)$ .

a)  $f(x) = \ln(x^2 - 1)$

b)  $f(x) = (\ln x)^2$

c)  $f(x) = e^{-x^2}$

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3. Bestimme  $f'(x)$ .

a)  $f(x) = \sqrt{e^x + 1}$

b)  $f(x) = 2 \cdot e^{\sqrt{x}}$

c)  $f(x) = \ln(1 + \sqrt{x})$

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4. Bestimme  $f'(x)$ .

a)  $f(x) = 2 \cdot \sin\left(\frac{1}{x}\right)$

b)  $f(x) = \frac{2x}{(2x + 1)^2}$

c)  $f(x) = \frac{1}{(2x + 1)^4}$

d)  $f(x) = \sqrt{\frac{2x}{x-2}}$

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5. Bestimme  $f'(x)$ .

a)  $f(x) = 2x - \sqrt{1 - x^2}$

b)  $f(x) = \ln \frac{x-1}{1+x}$

c)  $f(x) = (2x + 1) \cdot e^{2x+1}$

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## Lösungen

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1. a)  $f(x) = (2x+1)^3 \Rightarrow f'(x) = 3 \cdot (2x+1)^2 \cdot 2 = 6 \cdot (2x+1)^2$

b)  $3f(x) = 2 \cdot (x+1)^3 \Rightarrow f'(x) = 2 \cdot 3 \cdot (x+1)^2 = 6 \cdot (x+1)^2$

c)  $f(x) = 2 + (x+1)^3 \Rightarrow f'(x) = 0 + 3 \cdot (x+1)^2 \cdot 1 = 3 \cdot (x+1)^2$

d)  $f(x) = \sqrt[3]{\frac{1}{2}x+1} \Rightarrow f'(x) = \frac{1}{2 \cdot \sqrt[3]{\frac{1}{2}x+1}} \cdot \frac{1}{4} = \frac{1}{4 \cdot \sqrt[3]{\frac{1}{2}x+1}}$

e)  $f(x) = a \cdot \cos(bx+c) \Rightarrow f'(x) = a \cdot (-\sin(bx+c)) \cdot b = -ab \cdot \sin(bx+c)$

f)  $f(x) = 3 \cdot e^{1-2x} \Rightarrow f'(x) = 3 \cdot e^{1-2x} \cdot (-2) = -6 \cdot e^{1-2x}$

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2. Bestimme  $f'(x)$ .

a)  $f(x) = \ln(x^2-1) \Rightarrow f'(x) = \frac{1}{x^2-1} \cdot 2x = \frac{2x}{x^2-1}$

b)  $f(x) = (\ln x)^2 \Rightarrow f'(x) =$

c)  $f(x) = e^{-x^2} \Rightarrow f'(x) = e^{-x^2} \cdot (-2x) = -2x \cdot e^{-x^2}$

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3. Bestimme  $f'(x)$ .

a)  $f(x) = \sqrt{e^x+1} \Rightarrow f'(x) = \frac{1}{2\sqrt{e^x+1}} \cdot e^x = \frac{e^x}{2\sqrt{e^x+1}}$

b)  $f(x) = 2 \cdot e^{\sqrt{x}} \Rightarrow f'(x) = 2 \cdot e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \frac{e^{\sqrt{x}}}{\sqrt{x}}$

c)  $f(x) = \ln(1+\sqrt{x}) \Rightarrow f'(x) = \frac{1}{1+\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}(1+\sqrt{x})}$

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4. Bestimme  $f'(x)$ .

a)  $f(x) = 2 \cdot \sin\left(\frac{1}{x}\right) \Rightarrow f'(x) = 2 \cdot \sqrt{1-x^2} \cdot \left(-\frac{1}{x^2}\right) = -\frac{2}{x^2} \cdot \sqrt{1-x^2}$

$$\begin{aligned} \text{b) } f(x) &= \frac{2x}{(2x+1)^2} \Rightarrow f'(x) = \frac{2 \cdot (2x+1)^2 - 2x \cdot 2 \cdot (2x+1) \cdot 2}{(2x+1)^4} = \frac{2 \cdot (2x+1) - 2x \cdot 2 \cdot 2}{(2x+1)^3} = \\ &= \frac{4x+2-8x}{(2x+1)^3} = \frac{2-4x}{(2x+1)^3} \end{aligned}$$

$$\begin{aligned} \text{c) } f(x) &= \frac{1}{(2x+1)^4} = (2x+1)^{-4} \Rightarrow f'(x) = -4 \cdot (2x+1)^{-5} \cdot 2 = -8 \cdot (2x+1)^{-5} = \\ &= \frac{-8}{(2x+1)^5} \end{aligned}$$

$$\begin{aligned} \text{d) } f(x) &= \sqrt{\frac{2x}{x-2}} \Rightarrow f'(x) = \frac{1}{2\sqrt{\frac{2x}{x-2}}} \cdot \frac{1 \cdot (x-2) - 2x \cdot 1}{(x-2)^2} = \frac{1}{2} \sqrt{\frac{x-2}{2x}} \cdot \frac{-x-2}{(x-2)^2} = \\ &\frac{1}{2} \sqrt{\frac{x-2}{2x}} \cdot \frac{-x-2}{(x-2)^2} = -\frac{1}{2} \frac{x+2}{(x-2)^2} \cdot \sqrt{\frac{x-2}{2x}} \end{aligned}$$


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5. Bestimme  $f'(x)$ .

$$\text{a) } f(x) = 2x - \sqrt{1-x^2} \Rightarrow f'(x) = 2 - \frac{1}{2\sqrt{1-x^2}} \cdot (-2x) = 1 + \frac{x}{\sqrt{1-x^2}}$$

$$\text{b) } f(x) = \ln \frac{x-1}{x+1} \Rightarrow f'(x) = \frac{1}{\frac{x-1}{x+1}} \cdot \frac{1 \cdot (x+1) - (x-1) \cdot 1}{(x+1)^2} = \frac{x+1}{x-1} \cdot \frac{2}{(x+1)^2} = \frac{2}{(x-1) \cdot (x+1)}$$

$$\text{c) } f(x) = (2x+1) \cdot e^{2x+1} \Rightarrow f'(x) = 2 \cdot e^{2x+1} + (2x+1) \cdot e^{2x+1} \cdot 2 = (4x+4) \cdot e^{2x+1}$$


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