**Übungen : Potenzen mit rationalen Exponenten**

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1. Bestimme die L in G = ℝ :

a) $\frac{1}{2}x^4 - 20 = 12 \quad \Rightarrow \quad x = -4 \vee x = 4$

b) $2x^3 + 0{,}25 = 0 \quad \Rightarrow \quad x = -0{,}5$

c) $(x-4)^{-3} = 2 \quad \Rightarrow \quad \frac{1}{(x-4)^3} = 2 \quad \Rightarrow \quad \frac{1}{2} = (x-4)^3 \quad \Rightarrow \quad x - 4 = \sqrt[3]{0{,}5}$

$\Rightarrow \quad x = 4 + \sqrt[3]{0{,}5}$

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***Vereinfache jeweils ohne TR***

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2. a) $5^{\frac{1}{2}} \cdot 5^{\frac{1}{4}} = 5^{\frac{3}{4}} = 5^{\frac{3}{4}} = \sqrt[4]{125}$

b) $4^{-\frac{2}{3}} \cdot 4^{\frac{3}{4}} = 4^{\frac{1}{12}} = 2^{\frac{1}{6}} = \sqrt[6]{2}$

c) $2^{\frac{3}{5}} \cdot 2^{-\frac{3}{10}} = 2^{\frac{3}{10}} = \sqrt[10]{8}$

d) $125^{\frac{5}{6}} \cdot 125^{0{,}5} = 125^{\frac{4}{3}} = (\sqrt[3]{125})^4 = 625$

e) $256^{\frac{1}{12}} \cdot 256^{\frac{7}{24}} \cdot 256^{-\frac{1}{4}} = 256^{\frac{1}{8}} = 2$

f) $8^{-\frac{1}{2}} \cdot 4^{-\frac{1}{6}} = 2^{-\frac{3}{2}} \cdot 2^{-\frac{1}{3}} = 2^{-\frac{11}{6}} = 2^{-2+\frac{1}{6}} = 2^{-2} \cdot 2^{\frac{1}{6}} = \frac{\sqrt[6]{2}}{4}$

g) $10^{\frac{1}{2}} : 10^{\frac{1}{3}} = 10^{\frac{1}{6}} = \sqrt[6]{10}$

i) $2^{-\frac{2}{3}} : 2^{-0{,}5} = 2^{-\frac{1}{6}} = \frac{1}{\sqrt[6]{2}}$

k) $\frac{9^{\frac{2}{3}}}{9^{\frac{1}{6}}} = 9^{\frac{1}{2}} = 3$

l) $216^{-\frac{1}{4}} : 216^{\frac{5}{12}} = 216^{-\frac{2}{3}} = \frac{1}{36}$

m) $\frac{32^{0{,}3}}{32^{0{,}5}} = 32^{-0{,}2} = 32^{-\frac{1}{5}} = \frac{1}{\sqrt[5]{32}} = \frac{1}{2}$

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3. a) $a^{\frac{5}{3}} \cdot a^{\frac{1}{2}} = a^{\frac{13}{6}} = a^2 \cdot \sqrt[6]{a}$

b) $b^{-0{,}25} \cdot b^{\frac{1}{3}} = b^{\frac{1}{12}} = \sqrt[12]{b}$

c) $c^{-2} : c = c^{-3}$

d) $\frac{d^{-2}}{d^{\frac{3}{5}}} = d^{-2-\frac{3}{5}} = d^{-3+\frac{2}{5}} = \frac{\sqrt[5]{d^2}}{d^3}$

e) $e^{-\frac{1}{n}} \cdot e^{-\frac{1}{n}} = e^{-\frac{2}{n}} = \frac{1}{\sqrt[n]{e^2}}$

f) $f^{-\frac{1}{n}} : f^{\frac{1}{2n}} = f^{-\frac{1}{n}-\frac{1}{2n}} = f^{-\frac{3}{2n}} = \frac{1}{\sqrt[2n]{f^3}}$

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5. a) $4^{\frac{2}{5}} \cdot 2^{\frac{2}{5}} = 8^{\frac{2}{5}} = 2^{\frac{6}{5}} = 2\cdot\sqrt[5]{2}$

b) $10^{-0,4} : 2^{-0,4} = 5^{-0,4} = \frac{1}{\sqrt[5]{25}}$

c) $x^{\frac{2}{3}} : (2x)^{\frac{2}{3}} = \left(\frac{x}{2x}\right)^{\frac{2}{3}} = \left(\frac{1}{2}\right)^{\frac{2}{3}} = \frac{1}{\sqrt[3]{4}}$ oder $\left(\frac{1}{2}\right)^{\frac{2}{3}} = \left(\frac{1}{2}\right)^{-1+\frac{1}{3}} = \left(\frac{1}{2}\right)^{-1}\cdot\left(\frac{1}{2}\right)^{\frac{1}{3}} = 2\sqrt[3]{0,5}$

d) $(6x)^{\frac{1}{3}} \cdot (\frac{1}{x^2})^{\frac{1}{3}} = \left(6x\cdot\frac{1}{x^2}\right)^{\frac{1}{3}} = \left(\frac{6}{x}\right)^{\frac{1}{3}} = \sqrt[3]{\frac{6}{x}}$

e) $(2x)^{-\frac{1}{3}} \cdot (\frac{x}{2})^{\frac{1}{3}} = \frac{1}{(2x)^{\frac{1}{3}}}\cdot(\frac{x}{2})^{\frac{1}{3}} = \left(\frac{x}{2} : 2x\right)^{\frac{1}{3}} = \left(\frac{1}{4}\right)^{\frac{1}{3}} = \sqrt[3]{0,25}$

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6. a) $(2^{\frac{1}{3}})^6 = 2^2 = 4$

b) $(25^{\frac{3}{4}})^{-2} = 25^{-\frac{3}{2}} = \frac{1}{(\sqrt{25})^3} = \frac{1}{125}$

c) $(243^{-\frac{3}{4}})^{-1,6} = 243^{1,2} = 243^{\frac{6}{5}} = 729$

d) $(x^{-\frac{2}{3}})^{-\frac{3}{8}} = x^{\frac{1}{4}} = \sqrt[4]{x}$

e) $(2^{-\frac{1}{n}})^n = 2^{-1} = \frac{1}{2}$

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7. Gib das Ergebnis als Wuzel an

a) $\sqrt[3]{4} \cdot \sqrt[4]{4} = 4^{\frac{1}{3}}\cdot 4^{\frac{1}{4}} = 4^{\frac{7}{12}} = 2^{\frac{7}{6}} = 2\cdot\sqrt[6]{2}$

b) $\sqrt[5]{3} : \sqrt{3} = 3^{\frac{1}{5}} : 3^{\frac{1}{2}} = 3^{-\frac{3}{10}} = \frac{1}{\sqrt[10]{27}} = \sqrt[10]{\frac{1}{27}}$

c) $\sqrt[4]{2^9} \cdot \sqrt{2^9} = 2^{\frac{9}{4}} \cdot 2^{\frac{9}{2}} = 2^{\frac{27}{4}} = 2^{6+\frac{3}{4}} = 2^6\cdot\sqrt[4]{2^3} = 64\cdot\sqrt[4]{8}$

d) $\sqrt[3]{2} \cdot \sqrt[3]{4} = 2^{\frac{1}{3}}\cdot 4^{\frac{1}{3}} = 8^{\frac{1}{3}} = 2$

e) $\sqrt[4]{10x} : \sqrt[4]{2x} = (10x)^{\frac{1}{4}} : (2x)^{\frac{1}{4}} = 5^{\frac{1}{4}} = \sqrt[4]{5}$

f) $\sqrt[3]{\sqrt[3]{5}} = (5^{\frac{1}{3}})^{\frac{1}{3}} = 5^{\frac{1}{9}} = \sqrt[9]{5}$

g) $\sqrt[n]{\sqrt[3]{a}} = (a^{\frac{1}{3}})^{\frac{1}{n}} = a^{\frac{1}{3n}} = \sqrt[3n]{a}$

h) $\frac{\sqrt{b}\cdot\sqrt[3]{b}}{\sqrt[4]{b^3}} = \frac{b^{\frac{1}{2}}\cdot b^{\frac{1}{3}}}{b^{\frac{3}{4}}} = b^{\frac{1}{2}+\frac{1}{3}-\frac{3}{4}} = b^{\frac{1}{12}} = \sqrt[12]{b}$

i) $\frac{\sqrt[6]{2x} : \sqrt[3]{x}}{\sqrt{x} : \sqrt[4]{x}} = \frac{(2x)^{\frac{1}{6}} : x^{\frac{1}{3}}}{x^{\frac{1}{2}} : x^{\frac{1}{4}}} = \frac{2^{\frac{1}{6}}\cdot x^{\frac{1}{6}} : x^{\frac{1}{3}}}{x^{\frac{1}{2}-\frac{1}{4}}} = \frac{2^{\frac{1}{6}}\cdot x^{-\frac{1}{6}}}{x^{\frac{1}{4}}} = 2^{\frac{1}{6}}x^{-\frac{1}{6}-\frac{1}{4}} = 2^{\frac{1}{6}}x^{-\frac{5}{12}} =$

$= 2^{\frac{2}{12}}x^{-\frac{5}{12}} = \sqrt[12]{2^2x^{-5}} = \sqrt[12]{\frac{4}{x^5}}$

k) $\frac{y}{\sqrt[3]{y^2}\cdot\sqrt[4]{y}} = \frac{y}{y^{\frac{2}{3}}\cdot y^{\frac{1}{4}}} = \frac{y}{y^{\frac{11}{12}}} = y^{\frac{1}{12}} = \sqrt[12]{y}$

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8. Radiziere teilweise :

a) a) $\sqrt[3]{6^4} = 6^{\frac{4}{3}} = 6^{1+\frac{1}{3}} = 6\cdot\sqrt[3]{6}$

b) $\sqrt[3]{0{,}5^7} = 0{,}5^{\frac{7}{3}} = 0{,}5^{2+\frac{1}{3}} = 0{,}5^2\cdot 0{,}5^{\frac{1}{3}} = 0{,}25\cdot\sqrt[3]{0{,}5}$

c) $\sqrt[4]{x^8} = x^2$

d) $\frac{\sqrt[3]{40}}{2} = \frac{40^{\frac{1}{3}}}{2} = \frac{(8\cdot 5)^{\frac{1}{3}}}{2} = \frac{8^{\frac{1}{3}}\cdot 5^{\frac{1}{3}}}{2} = \sqrt[3]{5}$

e) $\frac{15}{\sqrt[3]{500}} = \frac{15}{\sqrt[3]{125\cdot 4}} = \frac{15}{5\cdot\sqrt[3]{4}} = \frac{3}{\sqrt[3]{4}}$

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9. Mache den Nenner rational :

a) $\frac{1}{\sqrt[3]{2}} = \frac{1}{2^{\frac{1}{3}}} = \frac{1\cdot 2^{\frac{2}{3}}}{2^{\frac{1}{3}}\cdot 2^{\frac{2}{3}}} = \frac{\sqrt[3]{4}}{2}$

b) $\frac{2}{\sqrt[4]{4}} = \frac{2}{4^{\frac{1}{4}}} = \frac{2}{2^{\frac{1}{2}}} = \frac{2}{\sqrt{2}} = \sqrt{2}$

c) $\frac{2}{\sqrt[3]{5^2}} = \frac{2}{5^{\frac{2}{3}}} = \frac{2\cdot 5^{\frac{1}{3}}}{5} = 0{,}4\cdot\sqrt[3]{5}$

d) $\sqrt[5]{\frac{2}{3}} = \frac{\sqrt[5]{2}}{\sqrt[5]{3}} = \frac{2^{\frac{1}{5}}}{3^{\frac{1}{5}}} = \frac{2^{\frac{1}{5}}\cdot 3^{\frac{4}{5}}}{3^{\frac{1}{5}}\cdot 3^{\frac{4}{5}}} = \frac{2^{\frac{1}{5}}\cdot(3^4)^{\frac{1}{5}}}{3} = \frac{\sqrt[5]{2\cdot 3^4}}{3} = \frac{\sqrt[5]{162}}{3}$

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10. Berechne bzw. vereinfache :

a) $\sqrt[3]{2a^2}\cdot\sqrt[3]{32a} = (2a^2)^{\frac{1}{3}}\cdot(32a)^{\frac{1}{3}} = (64a^3)^{\frac{1}{3}} = 4a$

b) $\sqrt{b}\cdot\sqrt[5]{b^{-2}}\cdot\sqrt[10]{b} = b^{\frac{1}{2}}\cdot b^{-\frac{2}{5}}\cdot b^{\frac{1}{10}} = b^{\frac{1}{5}} = \sqrt[5]{b}$

c) $\sqrt[3]{2c^2\sqrt{2c}}\cdot\sqrt{2c\sqrt[4]{2c^3}} = [2c^2\cdot(2c)^{\frac{1}{2}}]^{\frac{1}{3}}\cdot[2c\cdot(2c^3)^{\frac{1}{4}}]^{\frac{1}{2}} = [2c^2\cdot 2^{\frac{1}{2}}\cdot c^{\frac{1}{2}}]^{\frac{1}{3}}\cdot[2c\cdot 2^{\frac{1}{4}}c^{\frac{3}{4}}]^{\frac{1}{2}} =$

$= [2^{\frac{3}{2}}\cdot c^{\frac{5}{2}}]^{\frac{1}{3}}\cdot[2^{\frac{5}{4}}c^{\frac{7}{4}}]^{\frac{1}{2}} = 2^{\frac{1}{2}}\cdot c^{\frac{5}{6}}\cdot 2^{\frac{5}{8}}\cdot c^{\frac{7}{8}} = 2^{\frac{9}{8}}c^{\frac{41}{24}} = 2c\cdot\sqrt[24]{8c^{17}}$

d) $\left(\sqrt[6]{\frac{a^4c^5}{b^3}} : \sqrt[4]{\frac{a^2b}{c^3}}\right)\cdot\frac{\sqrt{b\sqrt{b}}}{\sqrt[4]{a^2c^5}} = \left(\frac{a^{\frac{2}{3}}c^{\frac{5}{6}}}{b^{\frac{1}{2}}} : \frac{a^{\frac{1}{2}}b^{\frac{1}{4}}}{c^{\frac{3}{4}}}\right)\cdot\frac{b^{\frac{1}{2}}b^{\frac{1}{4}}}{a^{\frac{1}{2}}c^{\frac{5}{4}}} = a^{-\frac{1}{3}}c^{\frac{1}{3}}$

e) $(a^{\frac{1}{3}} - 3a^{-\frac{1}{3}})^2 = a^{\frac{2}{3}} - 6 + a^{-\frac{2}{3}}$ ***Binomische Formel !***

f) $(a^{\frac{1}{2}} + a^{-\frac{1}{4}})(a^{\frac{1}{2}} - a^{-\frac{1}{4}}) = a - a^{-\frac{1}{2}}$ ***Binomische Formel!***

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**Testaufgaben**

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1. Vereinfache

a) $\sqrt[4]{2^6} : \left(\frac{1}{2}\right)^{-3} = 2^{\frac{6}{4}}\cdot 2^3 = 2^{\frac{9}{2}} = 16\sqrt{2}$

b) $(2b^3)^2 : (4b^{-4})^{\frac{1}{2}} = 4b^6 : (2b^{-2}) = 2b^8$

c) $\sqrt[3]{b^6} : \left(\frac{1}{27b^3}\right)^{-\frac{1}{3}} = b^2 : (27b^3)^{\frac{1}{3}} = b^2 : (3b) = \frac{b}{3}$

d) $\sqrt{\frac{a}{b}} \cdot \sqrt[3]{\frac{b^3}{a}} = (\frac{a}{b})^{\frac{1}{2}} \cdot (\frac{b^3}{a})^{\frac{1}{3}} = \frac{a^{\frac{1}{2}} \cdot b}{b^{\frac{1}{2}} \cdot a^{\frac{1}{3}}} = a^{\frac{1}{6}} b^{\frac{1}{2}}$

e) $\frac{(a^2)^{\frac{1}{3}} \cdot \sqrt{a^7}}{a^4 \cdot \sqrt[6]{a}} = \frac{a^{\frac{2}{3}} \cdot a^{\frac{7}{2}}}{a^4 \cdot a^{\frac{1}{6}}} = a^{\frac{2}{3}+\frac{7}{2}-4-\frac{1}{6}} = a^0 = 1$

f) $\sqrt[9]{x^6 \cdot \sqrt[4]{x^{12}}} = [x^6 \cdot x^3]^{\frac{1}{9}} = x$

g) $(3x^3)^3 : (9x^{-3})^{\frac{1}{3}} = 3^3x^3 : (9^{\frac{1}{3}} x^{-1}) = \frac{3^3x^3}{3^{\frac{2}{3}} x^{-1}} = 3^{\frac{7}{3}} x^4 = 9x^4 \cdot \sqrt[3]{3}$

h) $\left(\frac{\sqrt[12]{x^5}}{\sqrt[3]{x}} + \frac{\sqrt[4]{x}}{\sqrt[6]{x}}\right) \cdot \sqrt[12]{\frac{1}{x}} = \left(\frac{x^{\frac{5}{12}}}{x^{\frac{1}{3}}} + \frac{x^{\frac{1}{4}}}{x^{\frac{1}{6}}}\right) \cdot \left(\frac{1}{x}\right)^{\frac{1}{12}} = (x^{\frac{1}{12}} + x^{\frac{1}{12}}) \cdot \left(\frac{1}{x}\right)^{\frac{1}{12}} =$

$= 2x^{\frac{1}{12}} \cdot \left(\frac{1}{x}\right)^{\frac{1}{12}} = 2$

i) $\sqrt[4]{\frac{a^9}{\sqrt{a^3}}} - 2 \cdot \sqrt{a^3 \cdot \sqrt[4]{a^3}} = (\frac{a^9}{a^{\frac{3}{2}}})^{\frac{1}{4}} - 2 \cdot (a^3 \cdot a^{\frac{3}{4}})^{\frac{1}{2}} = a^{\frac{15}{8}} - 2a^{\frac{15}{8}} = -a^{\frac{15}{8}}$

j) $\sqrt{a \cdot \sqrt[3]{a}} \cdot \sqrt[3]{\sqrt{a^2}} \cdot \sqrt{\sqrt{\sqrt{a^{-4}}}} = (a \cdot a^{\frac{1}{3}})^{\frac{1}{2}} \cdot a^{\frac{1}{3}} \cdot a^{-\frac{1}{2}} = \sqrt{a}$

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2. Bestimme die Lösungsmenge in $G = \mathbb{R}$ :

$\left(17 - 3 \cdot \sqrt{5x-1}\right)^{\frac{1}{3}} = 2 \quad \Rightarrow \quad 17 - 3 \cdot \sqrt{5x-1} = 8 \quad \Rightarrow \quad -3 \cdot \sqrt{5x-1} = -9$

$\Rightarrow \quad \sqrt{5x-1} = 3 \quad \Rightarrow \quad 5x - 1 = 9 \quad \Rightarrow \quad x = 2$

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