

## Allgemeine Wurzel

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### Potenzschreibweise

$$a^{\frac{1}{n}} \cdot b^{\frac{1}{n}} = (a \cdot b)^{\frac{1}{n}}$$

$$a^{\frac{1}{n}} : b^{\frac{1}{n}} = \left(\frac{a}{b}\right)^{\frac{1}{n}}$$

$$\left(a^{\frac{1}{n}}\right)^{\frac{1}{m}} = a^{\frac{1}{nm}}$$

### Wurzelschreibweise

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$$

$$\sqrt[n]{a} : \sqrt[n]{b} = \sqrt[n]{\frac{a}{b}}$$

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[m \cdot n]{a}$$

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### 1. Radiziere teilweise

a)  $\sqrt[3]{54}$

b)  $\sqrt[5]{160}$

c)  $\sqrt[4]{160}$

d)  $\sqrt[6]{7290}$

e)  $\sqrt[4]{1250}$

f)  $\sqrt[6]{62,5}$

g)  $\sqrt[3]{1,715}$

h)  $\sqrt[5]{0,243}$

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### 2. Schreibe als einzige Wurzel

a)  $\sqrt[3]{2} : \sqrt[4]{3}$

b)  $\sqrt[3]{5} \cdot \sqrt{5}$

c)  $\sqrt{7} \cdot \sqrt[3]{7} \cdot \sqrt[6]{7}$

d)  $\sqrt{3} : \left( \sqrt[4]{2} : \sqrt[5]{\frac{2}{9}} \right)$

e)  $\sqrt[5]{a} \cdot \sqrt[10]{a^3}$

f)  $\sqrt[5]{b^{-5}} : \sqrt[4]{b^{-3}}$

g)  $\sqrt{c} \cdot \sqrt[5]{c^{-2}} \cdot \sqrt[10]{c}$

h)  $\sqrt{d} \cdot \sqrt[4]{d^3} : \sqrt[10]{d^0}$

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### 3. Schreibe als einzige Wurzel

a)  $\sqrt{2\sqrt{3}}$

b)  $\sqrt[3]{\frac{5}{\sqrt{5}}}$

c)  $\sqrt[3]{\sqrt[3]{3}}$

d)  $\sqrt{2 \cdot \sqrt[5]{\frac{1}{2} \sqrt[3]{4}}}$

e)  $\sqrt{a^3 \sqrt{a}}$

f)  $\sqrt[5]{b^{-4} \cdot \sqrt[4]{b}}$

g)  $\sqrt[4]{a^3 b \cdot \sqrt[5]{ab^3}}$

h)  $\sqrt{a^n \sqrt[3]{a^n} \sqrt[2]{a^n}}$ 

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## Lösungen

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$$1. a) \sqrt[3]{54} = \sqrt[3]{27 \cdot 2} = \sqrt[3]{27} \cdot \sqrt[3]{2} = 3\sqrt[3]{2}$$

$$b) \sqrt[5]{160} = \sqrt[5]{32 \cdot 5} = \sqrt[5]{32} \cdot \sqrt[5]{5} = 2\sqrt[5]{5}$$

$$c) \sqrt[4]{160} = \sqrt[4]{16 \cdot 10} = \sqrt[4]{16} \cdot \sqrt[4]{10} = 2\sqrt[4]{10}$$

$$d) \sqrt[6]{7290} = \sqrt[6]{729 \cdot 10} = \sqrt[6]{729} \cdot \sqrt[6]{10} = 3\sqrt[6]{10}$$

$$e) \sqrt[4]{1250} = \sqrt[4]{625 \cdot 2} = 5\sqrt[4]{2}$$

$$f) \sqrt{62,5} = \sqrt{6,25 \cdot 10} = 2,5\sqrt{10}$$

$$g) \sqrt[3]{1,715} = \sqrt[3]{\frac{343 \cdot 5}{1000}} = \frac{3}{10} \cdot \sqrt[3]{5}$$

$$h) \sqrt[5]{0,243} = \sqrt[5]{\frac{24300}{10000}} = 0,3 \cdot \sqrt[5]{100}$$

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$$2. a) \sqrt[3]{2} : \sqrt[4]{3} = 2^{\frac{1}{3}} : 3^{\frac{1}{4}} = 2^{\frac{4}{12}} : 3^{\frac{3}{12}} = (2^4)^{\frac{1}{12}} \cdot (3^3)^{\frac{1}{12}} = \sqrt[12]{2^4 \cdot 3^3} = \sqrt[12]{432}$$

$$b) \sqrt[3]{5} \cdot \sqrt{5} = 5^{\frac{1}{3}} \cdot 5^{\frac{1}{2}} = 5^{\frac{1}{3} + \frac{1}{2}} = 5^{\frac{5}{6}} = \sqrt[6]{5^5} = \sqrt[6]{3125}$$

$$c) \sqrt{7} \cdot \sqrt[3]{7} \cdot \sqrt[6]{7} = 7^{\frac{1}{2}} \cdot 7^{\frac{1}{3}} \cdot 7^{\frac{1}{6}} = 7^{\frac{1}{2} + \frac{1}{3} + \frac{1}{6}} = 7^1 = 7$$

$$\begin{aligned} d) \sqrt{3} : \left( \sqrt[4]{2} : \sqrt{\frac{2}{9}} \right) &= 3^{\frac{1}{2}} : \left[ 2^{\frac{1}{4}} : \left( \frac{2}{9} \right)^{\frac{1}{2}} \right] = 3^{\frac{1}{2}} : \left[ 2^{\frac{1}{4}} : \frac{2^{\frac{1}{2}}}{9^{\frac{1}{2}}} \right] = 3^{\frac{1}{2}} : \left[ 2^{\frac{1}{4}} \cdot \frac{3^{\frac{2}{5}}}{2^{\frac{1}{5}}} \right] = \\ &= 3^{\frac{1}{2}} : \left[ 2^{\frac{1}{20}} \cdot 3^{\frac{2}{5}} \right] = \frac{3^{\frac{1}{2}}}{2^{\frac{1}{20}} \cdot 3^{\frac{2}{5}}} = \frac{3^{\frac{1}{10}}}{2^{\frac{1}{20}}} = \frac{3^{\frac{2}{20}}}{2^{\frac{1}{20}}} = \sqrt[20]{\frac{3^2}{2}} = \sqrt[20]{\frac{9}{2}} = \sqrt[20]{4,5} \end{aligned}$$

$$e) \sqrt[5]{a} \cdot \sqrt[10]{a^3} = a^{\frac{1}{5}} \cdot a^{\frac{3}{10}} = a^{\frac{1}{5} + \frac{3}{10}} = a^{\frac{1}{2}} = \sqrt{a}$$

$$f) \sqrt[6]{b^{-5}} : \sqrt[4]{b^{-3}} = b^{-\frac{5}{6}} : b^{-\frac{3}{4}} = b^{-\frac{5}{6} - (-\frac{3}{4})} = b^{-\frac{1}{10}} = \sqrt[10]{b^{-1}} = \sqrt[10]{\frac{1}{b}}$$

$$g) \sqrt{c} \cdot \sqrt[5]{c^{-2}} \cdot \sqrt[10]{c} = c^{\frac{1}{2}} \cdot c^{-\frac{2}{5}} \cdot c^{\frac{1}{10}} = c^{\frac{1}{2} - \frac{2}{5} + \frac{1}{10}} = c^{\frac{1}{5}} = \sqrt[5]{c}$$

$$h) \sqrt{d} \cdot \sqrt[4]{d^3} : \sqrt[10]{d^0} = d^{\frac{1}{2}} \cdot d^{\frac{3}{4}} \cdot 1 = d^{\frac{5}{4}} = \sqrt[4]{d^5}$$


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$$3. a) \sqrt{2\sqrt{3}} = \sqrt{\sqrt{2^2 \cdot 3}} = \sqrt{\sqrt{12}} = \sqrt[4]{12}$$

$$b) \sqrt[3]{\sqrt{\frac{5}{\sqrt{5}}}} = \sqrt[3]{\sqrt{\sqrt{5}}} = (5^{\frac{1}{2}})^{\frac{1}{3}} = 5^{\frac{1}{6}} = \sqrt[6]{5}$$

$$c) \sqrt{3^3 \sqrt{3}} = \sqrt{\sqrt[3]{3^3 \cdot 3}} = \sqrt{\sqrt[3]{81}} = \sqrt[6]{81}$$

$$d) \sqrt{2 \cdot \sqrt[5]{\frac{1}{2} \sqrt[3]{4}}} = \sqrt{\sqrt[5]{2^5 \cdot \frac{1}{2} \cdot \sqrt[3]{4}}} = \sqrt{\sqrt[5]{16 \cdot \sqrt[3]{4}}} = \sqrt{\sqrt[5]{\sqrt[3]{16^3 \cdot 4}}} = \sqrt[30]{2^{14}} = \\ = \sqrt[15]{2^7} = \sqrt[15]{128}$$

oder

$$\sqrt{2 \cdot \sqrt[5]{\frac{1}{2} \sqrt[3]{4}}} = 2^{\frac{1}{2}} \cdot \left(\frac{1}{2}\right)^{\frac{1}{10}} \cdot 4^{\frac{1}{30}} = 2^{\frac{1}{2}} \cdot 2^{-\frac{1}{10}} \cdot 2^{\frac{1}{15}} = 2^{\frac{7}{15}}$$

$$e) \sqrt{a^3 \sqrt{a}} = a^{\frac{1}{2}} \cdot a^{\frac{1}{6}} = a^{\frac{2}{3}} = \sqrt[3]{a^2}$$

$$f) \sqrt[5]{b^{-4} \cdot \sqrt[4]{b}} = b^{-\frac{4}{5}} \cdot b^{\frac{1}{20}} = b^{-\frac{3}{4}} = \sqrt[4]{b^{-3}} = \sqrt[4]{\frac{1}{b^3}}$$

$$g) \sqrt[4]{a^3 b} \cdot \sqrt[5]{ab^3} = a^{\frac{3}{4}} b^{\frac{1}{4}} \cdot a^{\frac{1}{5}} b^{\frac{3}{5}} = a^{\frac{4}{5}} b^{\frac{2}{5}} = \sqrt[5]{a^4 b^2}$$

$$h) \sqrt{a^n \sqrt[3]{a^n \sqrt[2]{a^n}}} = a^{\frac{n}{2}} \cdot a^{\frac{n}{6}} \cdot a^{\frac{n}{12}} = a^{\frac{3n}{4}} = \sqrt[4]{a^{3n}}$$


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