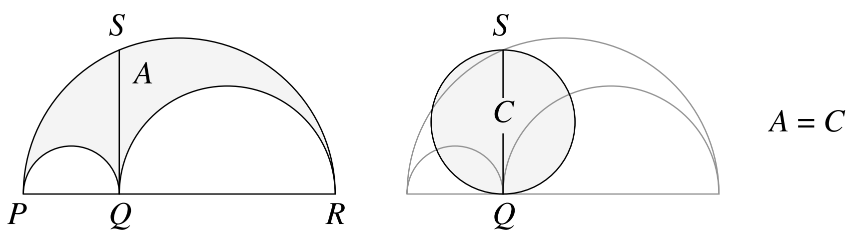
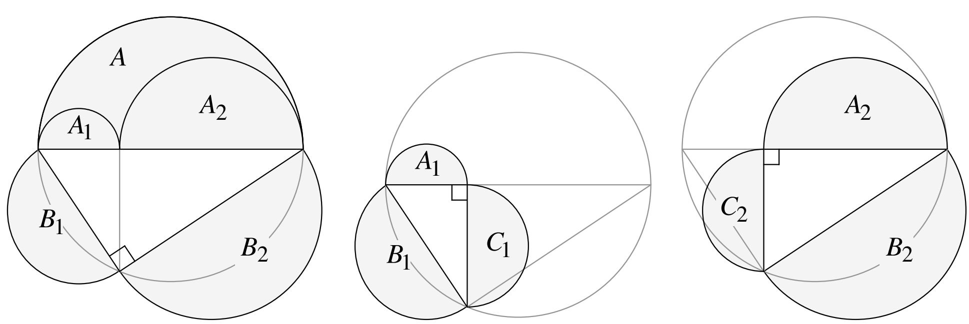
Proof Without Words: The Area of an Arbelos

THEOREM. Let P, Q, and R be three points on a line, with Q lying between P and R. Semicircles are drawn on the same side of the line with with diameters PQ, QR, and PR. An *arbelos* is the figure bounded by these three semicircles. Draw the perpendicular to PR at Q, meeting the largest semicircle at S. Then the area A of the arbelos equals the area C of the circle with diameter QS [Archimedes, *Liber Assumptorum*, Proposition 4].



Proof.



$A + A_1 + A_2 = B_1 + B_2$ $\quad$ $B_1 = A_1 + C_1$ $\quad$ $B_2 + A_2 + C_2$

$$A + A_1 + A_2 = A_1 + C_1 + A_2 + C_2$$
$$\therefore A = C_1 + C_2 = C$$

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